The Minimum Wage and Occupational Mobility

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Abstract

This paper quantifies the effect of the minimum wage on workers' occupational mobility. I construct a search-and-matching model with heterogeneous occupations and workers and highlight two channels by which the minimum wage decreases occupational mobility. First, it decreases vacancy posting and hence job arrival rate. Second, it compresses wages and therefore reduces the gain from switching, leading to a decrease in occupational mobility and an increase in mismatch. I show empirical evidence that the minimum wage decreases occupational mobility and increases mismatch. I calibrate the model to the US economy. The results suggest that a $15 minimum wage can damp aggregate output by as much as 0.4 percent, of which the wage compression channel accounts for 80 percent. My work shows a novel channel by which the minimum wage can decrease aggregate output, even if employment does not decrease.

Keywords: minimum wage, occupational mobility, mismatch, aggregate output.

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1 Introduction

In recent years, many states, counties, and cities have imposed minimum wage increases. The increases spurred ongoing debate on the potential effects of the minimum wage.[1] In this paper, I study the effect of the minimum wage on occupational mobility. By examining how the minimum wage affects workers' search incentives, I identify a novel response to minimum wage increases, namely occupational mobility. I also explore the implications of the mobility response on occupational mismatch and aggregate output.

I construct a search-and-matching model with heterogeneous occupations and workers. The model defines an occupation as a vector of skill composition and a worker as a vector of ability to learn those skills. Mismatch is the "distance" between an occupation and a worker. Firms post vacancies and form matches with workers. Productivity is match-specific and evolves stochastically. Productivity is more likely to increase when workers have a higher ability. It is more likely to decrease when mismatch is large. Occupational mobility is the rate at which employed workers switch occupations.

I highlight two channels by which the minimum wage affects occupational mobility. The first channel is that the minimum wage decreases firms' vacancy posting, hence job arrival rate. I refer to it as the employment effect channel. The decrease in vacancy posting reduces the occupational mobility of all workers, regardless of ability and mismatch.

The second channel is that the minimum wage narrows the wage gap between mismatched and better matched occupations. A higher minimum wage reduces the gain of switching occupations, which decreases workers' search incentives. I refer to it as the wage compression channel. The wage compression channel is more relevant for low ability workers because the minimum wage is more binding for them. By reducing workers' on-the-job search incentive, the wage compression channel restricts workers in mismatches longer, leading to an increase in mismatch.

I find two sets of empirical evidence of the model implications. First, I show that the minimum wage decreases occupational mobility. In particular, a 10% increase in the minimum wage decreases younger, less-educated workers' occupational mobility at the 4-digit Census code level by 3% in the Current Population Survey (CPS) from 2008 to 2016. I interpret the decrease in occupational mobility as a decrease in switching incentive rather than an increase in match quality.

Second, I show that the minimum wage positively correlates with mismatch using data from Guvenen et al. (2018). The data directly measure workers' mismatch, instead of using proxies such as wages or types of occupations. The result shows that the minimum

[1] See e.g. Card and Krueger (2015), Neumark and Wascher (2008).

wage positively correlates with younger, less-educated workers' mismatch, controlling for demographics and ability.[2]

I estimate the model using generalized method of moments (GMM) based on the empirical results. When the minimum wage increases from \$7.25 to \$15, the low ability workers' occupational mobility decreases by 44%. The decrease in mobility arises from the wage compression channel and displays non-linearity. The empirical wage distribution is flat at the current minimum wage level but then rises sharply. A small increase in the minimum wage would only affect a small fraction of the workers, while a large increase in minimum wage rapidly raises the fraction of workers with a binding minimum wage.

The increase in the minimum wage decreases aggregate output by 0.4%. The wage compression channel accounts for 80% of the decrease. The wage compression channel leads to more mismatch, which results in lower productivity. The reduction in output is from the low ability workers: their aggregate output decreases by 1.3%. To understand the large effect, note that the fraction of workers affected is substantial: around 40% of the labor force would face a binding minimum wage after the increase.[3]

This paper makes several contributions to the literature. First, my search-and-matching model extends the work-horse models in Moscarini (2005) and Flinn (2006) by incorporating heterogeneous occupations and workers. Similar to Moscarini (2005) and Flinn (2006), search friction and match-specific productivity create rents, and workers do not extract all the rents, giving rise to firms' monopsony power. The minimum wage *ex post* changes the "effective" monopsony power of the firms. By acting as a redistribution device, the minimum wage can have a minimal employment effect in the model (see e.g. Neumark (2018), Allegretto et al. (2017), Cengiz et al. (2019), Clemens and Wither (2019), Meer and West (2016), Harasztosi and Lindner (2019), Kreiner et al. (Forthcoming)). Introducing heterogeneous occupations allows me to study a new dimension of the effect of the minimum wage, as workers respond to an increase in the minimum wage with less occupational switching, leading to an efficiency loss.

Second, the paper contributes to the literature on price control and search behavior. I extend the static model in Fershtman and Fishman (1994) into a dynamic one with endogenous wage distributions. Fershtman and Fishman (1994) theoretically show that when search is costly, the minimum wage compresses the wage distribution, reducing the gain of searching and hence search effort. My model links search effort with match-specific

[2] I thank David Wiczer for kindly sharing the data in Guvenen et al. (2018), in which occupation, ability, and mismatch are defined identically as in my model.

[3] The estimate comes from projecting wages into 2020 on a 2% annual growth rate then imposing a \$15 minimum wage, using CPS 2008 to 2016 data.

productivity and endogenous wage distributions. By compressing endogenous wage distributions, the minimum wage disconnects search effort from match-specific productivity, decreasing workers' occupational switching which could otherwise result in a higher productivity. The minimum wage therefore shifts the wage distribution to the left, leading to a smaller reduction in wage inequality. The implications have empirical support from Clemens and Wither (2019) who show that wage growth is slower for low-skill workers in states where the federal minimum wage has deeper bite, and Autor et al. (2016) who show that the reduction in wage inequality by the minimum wage is smaller than previously estimated.

Empirically, to the best of my knowledge, I provide the first evidence of the negative correlation between the minimum wage and occupational mobility and the positive correlation between the minimum wage and mismatch. The result is important because while several papers show that the minimum wage increases job tenure (see e.g. Dube et al. (2007), Dube et al. (2016), Jardim et al. (2018)), different interpretations can lead to opposite implications. Dube et al. (2007) and Dube et al. (2016) interpret the positive correlation as a sign of an improvement in match quality. If the reasoning is true, one should observe a negative correlation between the minimum wage and mismatch because the minimum wage makes good matches last longer or increases the share of good matches. Instead, if the minimum wage increases job tenure by reducing workers' search incentive, it leads to slower labor market dynamism and more mismatch. The positive correlation between the minimum wage and mismatch supports the latter argument.

The results have important policy implications. The wage compression effect suggests that the minimum wage can lead to a non-trivial decrease in aggregate output, even if employment does not decrease.

Outline The organization of the rest of the paper is: section 2 lays out the model and derives the stationary equilibrium. I show that the minimum wage dis-incentivizes occupational switching in section 3. I present the empirical evidence of the model implications in section 4. In section 5 I estimate the model and quantitatively study the implications of the effect of the minimum wage on occupational mobility and aggregate output. Section 6 concludes. All proofs are in the appendix section B.

2 A Model with Heterogeneous Occupations and Workers

I construct a continuous-time search-and-matching model. The model features heterogeneous occupations and heterogeneous workers. It defines occupations as in Guvenen et al. (2018): an occupation is a vector $\boldsymbol{j} \in \mathbb{T}^n$ such that $\mathbb{T} = [0,1]$. Each dimension of the vector $\boldsymbol{j}$ corresponds to a specific skill, with its value representing the importance of that skill. An occupation differs from another one by having different compositions of skill importance. A unit measure of workers differ ex ante in a multidimensional vector called "ability": $\boldsymbol{a} \in \mathbb{T}^n$ in which $\mathbb{T} = [0,1]$. Each dimension of the ability determines the rate at which the workers learn the corresponding skill. The CDF of occupations is $H(\boldsymbol{j})$. The density of occupations is $h(\boldsymbol{j})$. The distribution of the workers' ability is given by the CDF $G(\boldsymbol{a})$ with PDF $g(\boldsymbol{a})$.[4]

To help understand the definition of occupations and workers, consider the following example. An occupation is a two-dimensional vector, (s_1, s_2), with the first dimension representing verbal skill and the second dimension representing social skill. Thus, the occupation "food server" would be denoted by $(0.3, 0.6)$ while the occupation "office clerk" would be represented by $(0.7, 0.4)$. If a worker is $(0.5, 0.9)$, it means that the worker's ability to learn verbal skills is 0.5 and 0.9 for social skill. Her mismatch in "food server" is 0.36 and 0.54 in "office clerk". She is better matched at "food server", which is consistent with social skills being more important in the occupation and her higher ability to learn social skills.

Firms post vacancies with a flow cost κ. The distributions of occupations $H(\boldsymbol{j})$ and workers $G(\boldsymbol{a})$ determine the occupation of the vacancy and the type of workers the vacancy meets. Once a match forms, the vacancy's outside option is normalized to be 0.

An employed worker with ability vector $\boldsymbol{a}$ in an occupation with skills composition vector $\boldsymbol{j}$ can produce a single good in a match. Let $(\Omega, \mathscr{F}, \mathbb{P}, \{\mathscr{F}_t : t \geqslant 0\})$ denote the filtered space associated with the match. The productivity of a worker is match-specific and stochastic, because of factors such as learning-by-doing, match-specific human capital accumulation.

Productivity evolves depending on the quality of the match. Define mismatch as $||\boldsymbol{a} - \boldsymbol{j}||$ in which $||\cdot||$ is the Euclidean norm in $\mathbb{T}^n$. It captures the degree to which the occupation's skill composition and the worker's ability to learn those skills overlap. A smaller mismatch means that the worker's ability is more suited to learning the skills of the occupation. As a result, the worker's match-specific productivity is more likely to rise. A larger mismatch means that productivity evolution is more likely to decline.

Besides mismatch, the evolution of productivity also depends on the worker's "absolute advantage" $||\boldsymbol{a}||$. There is a larger probability for productivity to increase under larger $||\boldsymbol{a}||$.

[4] All results are retained if I have a joint CDF $N(\boldsymbol{a}, \boldsymbol{j})$. I define $H(\boldsymbol{j})$ and $G(\boldsymbol{a})$ separately for ease of exposition.

Concretely, an $\boldsymbol{a}$ worker in occupation $\boldsymbol{j}$ has productivity as a stochastic process $\{X_t\}_{t\geqslant 0}$ that satisfies the following stochastic differential equation (SDE):

$$dX_t = \tilde{a}X_t dt + \sigma X_t dZ_t \tag{1}$$

in which $\{Z_t\}_{t\geqslant 0}$ is a Brownian motion on the filtered space.

I make the normalization that productivity maps one-to-one to output flow. I use productivity, current output, or simply output interchangeably, with the understanding that all three are flow values.

The drift of equation (1) is increasing in ability $||\boldsymbol{a}||$ and decreasing in mismatch $||\boldsymbol{a} - \boldsymbol{j}||$. The formulation compactly captures the relation between productivity, ability, and mismatch. First, fixing $\boldsymbol{a}$, $\tilde{a}$ is decreasing in mismatch $||\boldsymbol{a} - \boldsymbol{j}||$. A larger mismatch leads to a higher probability of downward trajectories of productivity. If $||\boldsymbol{a} - \boldsymbol{j}||$ is large, the match-specific productivity X_t converges to 0 in expectation. Fixing $||\boldsymbol{a}-\boldsymbol{j}||$, $\tilde{a}$ is increasing in $||\boldsymbol{a}||$, so that workers with a higher ability to learn are more likely to perform well in the match. If $\tilde{a}$ is large, the match-specific productivity increases on average. I impose an upper-bound $\overline{x}$ on the match-specific productivity X_t so it cannot be arbitrarily large.[5]

Unemployed workers receive unemployment benefit b and search for jobs with the arrival rate λ determined in equilibrium. Upon job arrival, they are matched into one of the occupations with initial productivity x_p. Employed workers can search on the job with efficiency given by α, so that job arrival rate on the job is $\alpha\lambda$.

Upon switching occupation, the workers need to pay a onetime cost ϕ. Workers are exogenously separated into unemployment at rate δ. Workers and firms are risk neutral. They share the common discount parameter r. Other than the switching cost, search is costless.

Search friction and match-specific productivity create rents for each realized match. The workers and firms split the surplus according to the generalized Nash bargaining, in which the workers have bargaining power β.[6] The Nash bargaining is subject to the minimum wage constraint as in Flinn (2006).

Because a worker is risk neutral, her objective is to maximize expected discounted wages. Let w be the wage function that realizes the bargained outcome. The worker with

[5] I micro-found the productivity process in a Ben-Porath economy in appendix section E.

[6] See Moscarini (2005) for a detailed discussion of how the combination of poaching auction and costless searching lead to the Nash bargaining wage setting with on-the-job search. I follow the argument in this paper.

ability $\boldsymbol{a}$ on occupation $\boldsymbol{j}$ maximizes:

$$V(x, \boldsymbol{a}, \boldsymbol{j}, m) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt} w(X_t, m) dt\right]$$

in which x is the productivity at time 0 and m is the minimum wage.[7] The value function depends on the worker's ability and her occupation because the output process is a function of the two variables. It also depends on the minimum wage m. In the rest of paper, I abstract from the dependence and write $V(x)$ for simplicity unless confusion arises. The worker's only choices are whether to search on the job and whether to separate endogenously into unemployment.[8]

For an unemployed worker with ability $\boldsymbol{a}$, the value function is

$$U(\boldsymbol{a}, m) = b + \lambda \mathbb{E}[V(x, \boldsymbol{a}, \boldsymbol{j}, m)]$$

in which the expectation is with respect to the output process and the occupation distribution. When there is no confusion, I write U in place of $U(\boldsymbol{a}, m)$. For the rest of the section, I focus on the stationary equilibrium.

2.1 Wage Function

Let us fix a worker-occupation match $(\boldsymbol{a}, \boldsymbol{j})$. Let $V(x)$ denote the worker's value function in which x is her productivity. The worker receives her instantaneous wage payment $w(x)$. She can search on the job in which job offers arrive at rate $\alpha\lambda$. If she accepts the offer, she pays the switching cost ϕ. Her initial output at her new occupation is x_p. The worker also faces exogenous separation at rate δ, upon which she is unemployed and receives the value of unemployment U. In the appendix, I derive the second-order differential equation that her value function follows:

$$\begin{aligned} rV(x) =& w(x) + \tilde{a} x V'(x) + \frac{1}{2}\sigma^2 x^2 V''(x) - \delta[V(x) - U] \\ &+ \alpha\lambda \mathbb{I}_{\left\{\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j}) dH(\boldsymbol{j}) - V(x) \geqslant \phi\right\}} \left[\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j}) dH(\boldsymbol{j}) - V(x) - \phi\right] \end{aligned} \tag{2}$$

[7] This formulation is for the ease of exposition. See equation (B.21) in appendix section B for the expanded formulation.

[8] Since search is costless, technically a worker can always search on the job. However, when her productivity is high, she would not switch even if she receives a job offer. My formulation has the advantage of being robust to a small perturbation to search cost.

$\mathbb{I}$ is the indicator function. The expected gain of on-the-job search is $\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j})dH(\boldsymbol{j}) - \phi$. The worker compares the expected gain with her current value on the job $V(x)$. Depending on productivity x, the worker might or might not search on the job.

If the worker is unemployed, she receives unemployment benefit b and searches for jobs. Her value of unemployment follows the equation:

$$rU = b + \lambda\left[\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j})dH(\boldsymbol{j}) - U\right] \tag{3}$$

While on the job, a worker can endogenously quit to unemployment. The worker faces a standard optimal switching problem. Let $\underline{x} = inf\{x : V(x) > U\}$. The worker will quit to unemployment endogenously if her productivity x is less than $\underline{x}$. I refer to $\underline{x}$ as the endogenous separation cutoff.

The value function of the matched vacancy is:

$$\begin{aligned} rJ(x) = x - w(x) + \tilde{a}xJ'(x) + \frac{1}{2}\sigma^2x^2J''(x) - \delta J(x) \\ - \alpha\lambda\mathbb{I}_{\left\{\int_{\mathbb{T}^n} V(x_p,\boldsymbol{j})dH(\boldsymbol{j})-V(x)\geqslant\phi\right\}}J(x) \end{aligned} \tag{4}$$

To simplify notation, I write $\mathbb{I}_{\left\{\int_{\mathbb{T}^n} V(x_p,\boldsymbol{j})dH(\boldsymbol{j})-V(x)\geqslant\phi\right\}}$ as $\mathbb{I}_{sw}$ in the remaining sections.

Because the surplus of the match is split by the generalized Nash bargaining, the worker's and the firm's value function are linearly related:

$$\beta J(x) = (1-\beta)[V(x) - U] \tag{5}$$

The relation equation (5) holds for all levels of productivity x. I seek classic solutions of equations (2) and (4), which implies that the linear relation holds for the derivatives of the value functions. The wage function is:

$$w(x) = \max\left\{\beta x + (1-\beta)b + \lambda(1-\beta)(1-\alpha\mathbb{I}_{sw})\left[\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j})dH(\boldsymbol{j}) - \phi - U\right], m\right\} \tag{6}$$

in which m is the minimum wage. The wage function in the absence of the minimum wage has three parts. The first and the second part suggest that the workers' wage is a convex combination of her current output and her unemployment flow benefit b. These two parts are standard in search models in which workers and firms share surplus linearly. The third part implies that the wage aggregates the worker's outside option given by $\lambda(1-\beta)\left[\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j})dH(\boldsymbol{j}) - \phi - U\right]$. The firm knows the worker's on-the-job search decision. Searching on the job incurs a wage cut of size $\lambda(1-\beta)\alpha\mathbb{I}_{sw}\left[\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j})dH(\boldsymbol{j}) - U - \phi\right]$.

2.2 Analytic Solution of Value Function

I set

$$x_s = inf\left\{x : V(x) = \int_{\mathbb{T}^n} V(x_p, \boldsymbol{j})dH(\boldsymbol{j}) - \phi\right\} \tag{7}$$

Similar to the endogenous separation cutoff, x_s determines the worker's on-the-job search decision. The worker will search on-the-job if and only if her current output is between $\underline{x}$ and x_s. Costless search makes a worker indifferent between searching and not searching when the output exceeds x_s. I stick to the formulation that the workers stop search when the output is above x_s. Besides being robust to a small perturbation to search cost, the formulation allows me to non-trivially derive the measure of workers who are searching.

The point x_s characterizes the worker's occupational switching decision. When the worker's productivity is between $\underline{x}$ and x_s, the expected payoff of staying in the current job is less than the expected payoff of switching occupation. The worker will search on-the-job for other occupations. Knowing x_s and $\underline{x}$ is equivalent to knowing the worker's occupation switching and endogenous separation decision. Given the worker's productivity path, I can calculate the fraction of time that the worker spends searching on the job. The occupational mobility rate is hence the average time the worker's productivity process drifts between $\underline{x}$ and x_s.

The following proposition characterizes firms' value function:

Proposition 1. *The value function $J(x)$ is log concave and has the form:*

$$J(x) = \begin{cases} C_0^0 x^{\gamma_0^0} + C_1^0 x^{\gamma_1^0} - A(x,m), & \textit{if } \underline{x} \leqslant x \leqslant x_s \\ C_0^1 x^{\gamma_0^1} + C_1^1 x^{\gamma_1^1} - B(x,m), & \textit{if } x > x_s \end{cases} \tag{8}$$

in which A and B are functions of productivity x and minimum wage m. The power coefficients γ_0^1, γ_1^0, γ_0^1, γ_1^1 are determined by model parameters and satisfy $\gamma_0^0, \gamma_0^1 < 0$, $\gamma_1^0, \gamma_1^1 > 0$.

The solution of the value function also includes six parameters determined by boundary conditions. Because I seek a solution that is twice continuously differentiable, I solve the parameters using the following conditions: $J(\underline{x}) = 0$, $J'(\underline{x}+) = 0$, $J(x_s-) = J(x_s+)$, $J'(x_s-) = J'(x_s+)$, $J''(x_s-) = J''(x_s+)$, $J(x_s) = A(x_s, m)$. The log-concavity of the value function validates the use of generalized Nash bargaining. The shock σ to the match-specific productivity affects the shape of the value function. As a result, σ affects the worker's occupational mobility by determining the cutoffs $\underline{x}$ and x_s.

Proposition 1 shows that when I fix the integral to be a constant in equation (2), I can solve for the twice-continuously differentiable value function. To guarantee that the whole system of solutions is consistent, I need to prove the existence of a fixed point. The fixed point is a function of $\boldsymbol{a}$. When one uses the function to solve for the family of solutions $\{V\}_{(\boldsymbol{a},\boldsymbol{j})}$, integrating the family of solutions on $\boldsymbol{j}$ would lead to the fixed point function. I leave the proof in the appendix section B.

Proposition 2. *There is a family of value functions $\{V, J, U\}_{(\boldsymbol{a},\boldsymbol{j})}$ that solves the family of ODEs equation (2) and equation (4).*

2.3 Stationary Wage Distribution

The worker's wage function is linear in her productivity x. The distribution of the wage and the productivity is thus the same up to relocation and rescaling. The upper-bound on the productivity process $\overline{x}$ guarantees a stationary distribution.[9]

The Fokker-Planck equation that the stationary productivity distribution satisfies on $(\underline{x}, \overline{x})$ is:

$$\frac{\sigma^2}{2}x^2 f''(x) + (2\sigma^2 - \tilde{a}^2)xf'(x) + (\sigma^2 - \tilde{a})f(x) - (\delta + \alpha\lambda\mathbb{I}_{\{x<x_s\}})f(x) = 0 \tag{9}$$

Equation (9) implies that the density flowing into and out of any interior point in $(\underline{x}, \overline{x})$ must be equal. Equation (9) holds regardless of the existence of the minimum wage. This is because the minimum wage does not affect the worker's productivity process equation (1). The existence of the cutoff points $\underline{x}$ and x_s is given by the optimal stopping problem which is also not affected by the minimum wage. The minimum wage only affects the locations of the cutoff points.

Equation (9) is a second-order ODE. Proposition 3 characterizes the stationary output distribution.

Proposition 3. *Assuming an reflecting upper-bound $\overline{x}$, the stationary output distribution is:*

$$f(x) = \begin{cases} B_0^0 x^{\eta_0^0} + B_1^0 x^{\eta_1^0}, & \text{if } \underline{x} \leqslant x \leqslant x_s \\ B_0^1 x^{\eta_0^1} + B_1^1 x^{\eta_0^1}, & \text{if } x > x_s \end{cases} \tag{10}$$

The parameters η_0^0, η_1^0, η_0^1, η_1^1 satisfy $\eta_0^0, \eta_0^1 < 0$, $\eta_1^0, \eta_1^1 > 0$.

At the lower-bound $\underline{x}$, the distribution $f(x)$ satisfies an absorbing boundary condition. The corresponding equation is $f(\underline{x}+) = 0$. The worker will quit to unemployment once

[9]There are several explanations for an upper-bound on productivity. I can make the upper-bound so large that the probability of reaching this upper-bound in finite time is small. Empirically, we also observe upper-bound of wages.

her productivity is below $\underline{x}$. At the upper-bound, the solution has an reflecting boundary condition by assumption. The corresponding equation is:

$$(\tilde{a} - \sigma^2) f(\overline{x}) = \frac{1}{2}\sigma^2 \overline{x} f'(\overline{x})$$

Two more conditions pin down the stationary distribution. The first is that total flow in and out of unemployment is constant. The second condition is that the total flow in and out of employment must balance. I leave the details of the conditions in the appendix.

The stationary distribution has a Pareto tail, consistent with the empirical evidence. The Pareto tail comes from randomness in productivity. It is locally decreasing in the productivity volatility σ: a larger volatility implies a thinner right tail and hence less inequality at the upper end. The intuition is that when the probability of large negative shock is high, a high productivity is more likely to become small. The Pareto tail is locally increasing in ability: there is a fatter right tail and hence more inequality among workers with higher ability.

2.4 Stationary Equilibrium: Definition and Existence

Let $f(x, \boldsymbol{a}, \boldsymbol{j}, m)$ be the stationary distribution of output associated with the pair $(\boldsymbol{a}, \boldsymbol{j})$ and the minimum wage m. Let s be the measure of job seekers. It is equal to the sum of unemployed workers and on-the-job searchers:

$$s = 1 - \int\int\int_{\underline{x}}^{\overline{x}} f(x, \boldsymbol{a}, \boldsymbol{j}, m) dx dG(\boldsymbol{a}) dH(\boldsymbol{j}) + \alpha \int\int\int_{\underline{x}}^{x_s} f(x, \boldsymbol{a}, \boldsymbol{j}, m) dx dG(\boldsymbol{a}) dH(\boldsymbol{j}) \quad (11)$$

Let v be the measure of vacancy. In equilibrium, I assume that the job arrival rate is determined by a constant return to scale, concave matching function $M(s, v) = s^{\zeta} v^{1-\zeta}, \zeta \in (0, 1)$. The job arrive rate is $\lambda = M(s, v)/s = M(1, \theta) = \theta^{1-\zeta}$ in which θ is the market tightness. By the free entry condition, the cost of a vacancy κ must equal to the expected gain of a firm:

$$\kappa = \int\int \lambda^{\frac{\zeta}{\zeta-1}} J(x_s, \boldsymbol{a}, \boldsymbol{j}, m) dG(\boldsymbol{a}) dH(\boldsymbol{j}) \quad (12)$$

A stationary general equilibrium is a set of parameters $\{\lambda, \underline{x}, s, v, \theta\}$ and a list of functions $\{J, V, f\}_{\boldsymbol{a},\boldsymbol{j}}$ which satisfy equations (8) and (10) to (12). For the existence of an equilibrium, I need a λ such that equation (12) holds for some $\kappa \in (0, +\infty)$. Since $J(x, \boldsymbol{a}, \boldsymbol{j}, m)$ is uniformly bounded in the interval $[J(\underline{x}, \mathbf{0}, \mathbf{1}, m), J(\overline{x}, \mathbf{1}, \mathbf{0}, m)]$ where $\mathbf{1}$ is the vector $(1, 1, ..., 1)$ and similarly for $\mathbf{0}$, the integrand on the right hand side of equation (12) decreases from $+\infty$ to 0 as λ increases from 0 to $+\infty$. Uniform boundedness of the function $J(x, \boldsymbol{a}, \boldsymbol{j}, m)$

also implies the continuity of the integral in λ. For any $\kappa \in (0, +\infty)$, there is at least one λ such that equation (12) holds, which guarantees the existence of a stationary general equilibrium:

Proposition 4. *There exists a stationary general equilibrium for any vacancy cost $\kappa \in (0, +\infty)$.*

2.5 Expression for Occupational Mobility

The occupational mobility in the economy is:

$$\mu = \alpha\lambda \int\int\int_{\underline{x}}^{x_s} f(x, \boldsymbol{a}, \boldsymbol{j}, m)dxdG(\boldsymbol{a})dH(\boldsymbol{j}) \tag{13}$$

It is the integral of the stationary output distribution between $\underline{x}$ and x_s, weighted by the ability and the occupation distribution.[10] This definition is consistent with the one in section 4 since both measure only on-the-job occupational switching.

For a fixed ability $\boldsymbol{a}$, the occupational mobility is:

$$\mu_{\boldsymbol{a}} = \alpha\lambda \int\int_{\underline{x}}^{x_s} f(x, \boldsymbol{a}, \boldsymbol{j}, m)dxdH(\boldsymbol{j}) \tag{14}$$

Equation (14) allows me to decompose how the occupational mobility responds to minimum wage increases. This is my goal in the next section.

3 The Effect of Minimum Wages on Occupational Mobility

The workers' occupational mobility depends on two factors: job arrival rate and switching profitability. Accordingly, the minimum wage affects occupational mobility by two channels: the employment effect channel and the wage compression channel.

3.1 The Employment Effect Channel

The minimum wage has a displacement effect: a higher minimum wage decreases firms' value of a match, leading to a larger probability of endogenous separation. The increasing separation decreases occupational mobility because workers who would have remained

[10] Kambourov and Manovskii (2004) notes that occupational mobility is correlated with wage inequality. This is also true in my model because both occupational mobility and wage inequality are determined by the same stationary distribution.

on the job and search for other occupations are now displaced.[11] The size of the effect depends not only on the magnitude of the minimum wage change but also on the probability that productivity drifts in low values. My model hence features differential displacement effects based on ability and mismatch.

The minimum wage also reduces firms' vacancy posting. By equation (12), firms adjust their vacancy postings in the new equilibrium when the minimum wage increases. The value function of firms decreases, so that firms reduce their vacancy posting. The reduction in vacancy posting results in a lower job arrival rate which decreases occupational mobility. Importantly, the reduction in vacancy posting affects all workers' occupational mobility regardless of their ability and mismatch.

3.2 The Wage Compression Channel

Besides the employment effect channel, the model shows that the minimum wage can decrease occupational mobility by a second channel because of wage compression. Proposition 5 is important to the wage compression channel:

Proposition 5. *Conditional on employment, the worker's value function is increasing in the minimum wage. The amount of the increase is decreasing in worker's ability* $||\boldsymbol{a}||$ *but increasing in mismatch* $||\boldsymbol{a} - \boldsymbol{j}||$.

Proposition 5 implies that the minimum wage has varying effects across the ability and mismatch distribution. High ability workers will benefit less from the minimum wage increase because the minimum wage rarely binds for them. When the minimum wage increases, there is little increase in their value function. One can view the value function as the integral of future potential wage paths, weighted by the probability of each path. For the high ability workers, it is rare for their wage paths to drop below the minimum wage. An increase in the minimum wage does not affect their value function by much.

Low ability workers are more likely to face a binding minimum wage. An increase in the minimum wage would increase their value function by much more. The minimum wage increases the wage on each affected wage path and the measure of wage paths with a binding minimum wage.

Similarly, mismatched workers benefit more from a minimum wage increase. The ones that benefit the most from a minimum wage increase are the low ability workers in mismatched occupations, conditional on staying employed. The distributional effect

[11] Mathematically, it means that the output at which J is zero would be higher than before after a minimum wage increase.

of the minimum wage on the workers' value function along two dimensions—ability and mismatch—is the key to the wage compression channel.[12]

To illustrate how the minimum wage can affect a worker's occupational mobility via the wage compression channel, consider a low ability worker in a mismatched occupation. I fix a pair $(\boldsymbol{a}, \boldsymbol{j})$ such that $||\boldsymbol{a}||$ is small and $||\boldsymbol{a} - \boldsymbol{j}||$ is large.

The intercept of the value function and the outside option determines her on-the-job search cutoff point, as shown in figure 1. Before the minimum wage, the cutoff point is the "old x_s" in the figure. By proposition 5, when the minimum wage increases, conditional on staying employed, she experiences a large increase in her value function. The value function shifts upward. By comparison, her outside option increases by less. This is because she would have less mismatch in her outside option, which according to proposition 5 would give her less gain when the minimum wage increases. The minimum wage increase narrows the wage gap between a mismatch and a good match. The new cutoff point "new x_s" moves to the left of the old cutoff point.

Figure 2 illustrates the effect of the leftward movement of on-the-job-search cutoff point on occupation mobility. Before the minimum wage increase, the low ability worker searches on the job for other occupations when her output is between $\underline{x}$ and x_s^{old}. Once her output exceeds x_s^{old}, she stops searching on the job. Figure 2 (a) shows her decision rules. Figure 2 (b) indicates what would happen when the minimum wage increases. The wage compression effect moves the on-the-job-search cutoff to the left. The on-the-job-search region shrinks. The probability of the worker's output being in the "search" region decreases. A decrease in occupational mobility follows.

The wage compression channel has differential effects on workers' occupational mobility: it is less relevant for high ability workers. This is because the expected output is increasing in ability, so that high ability workers are less likely to face a binding minimum wage. As a result, the minimum wage increase has much less effect on the high ability workers' occupational mobility.

Because the wage compression channel affects mismatched workers more, the minimum wage reduces their occupational mobility more. By restricting workers in mismatches longer, the wage compression channel implies that a higher minimum wage leads to more mismatch.

[12]While proposition 5 is intuitive, the proof requires reformulating the problem as an infinite horizon optimal switching problem. I leave the details in the appendix section B.

3.2.1 A Discussion of the Effect of the Minimum Wage on Search Effort and Labor Force Participation

There is literature that emphasizes the effect of minimum wages on increasing the search effort of unemployed workers (e.g. Acemoglu (2001), Flinn (2006), Ahn et al. (2011)). The model can extend to allow for such a response: let $e(m)$ denote the increase in search effort as a function of the minimum wage. The effective measure of job seekers, i.e. equation (11) becomes

$$\begin{aligned} s =& e(m)\left[1 - \int\int\int_{\underline{x}}^{\overline{x}} f(x, \boldsymbol{a}, \boldsymbol{j}, m) dx dG(\boldsymbol{a}) dH(\boldsymbol{j})\right] \\ &+ \alpha \int\int\int_{\underline{x}}^{x_s} f(x, \boldsymbol{a}, \boldsymbol{j}, m) dx dG(\boldsymbol{a}) dH(\boldsymbol{j}) \end{aligned} \tag{15}$$

The increase in search effort would counteract the decrease in vacancy posting. If it dominates, the job arrival rate could increase.

I note first that there is little empirical evidence of higher search effort after a minimum wage increase. Adams et al. (2018) find that recent minimum wage increases lead to a transitory increase in the search effort of unemployed workers. In the stationary equilibrium, such a transitory increase becomes irrelevant. More importantly, the wage compression channel is independent of the search effort response. It unambiguously reduces occupational mobility.

Another related issue is that I do not model labor force participation. If the minimum wage induces labor market entry of non-participants, the effective measure of job seekers could increase, leading to an increase in job arrival rate. It could also mean that the minimum wage can increase efficiency by ex post changing the bargaining power of the workers.[13] The model can accommodate the argument in a reduced-form way, since equation (15) also captures the increase in effective search effort because of labor market entry. A comprehensive analysis of the optimal level of minimum wage by incorporating labor force participation decisions is beyond the scope of the paper. I leave it for future extension.

4 Empirical Evidence

The model suggests that the minimum wage decreases low ability workers' incentive to switch occupations, leading to a decrease in occupational mobility and an increase in mismatch. I provide two sets of empirical evidence.

[13] This is the point made by Flinn (2006). The minimum wage has a similar effect in my model, but the welfare implication is less relevant because there is no labor force participation decision.

4.1 The Effect of the Minimum Wage on Occupational Mobility

I first estimate the effect of the minimum wage on occupational mobility. I use CPS 2008 to 2016 as my sample to analyze the effect of minimum wages on occupational mobility. I consider aggregate level analysis. In the appendix section A.1, I use micro-level data and different sample periods to corroborate the evidence.[14]

I merge two consecutive monthly files and drop the observations that are first or fifth month in the sample. I identify an occupational switch if a worker: 1. remains employed in both periods; 2. has different occupation codes across periods; 3. experience employer change in the second period.[15]

I identify an occupational stayer if a worker: 1. remains employed in both periods; 2. has the same occupational code in both periods; 3. has a non-empty entry in the employer switch question. This means that if a worker switches employer but stays in the same occupation, I still consider her an occupational stayer.

To construct the state-level occupational mobility rate, I aggregate the occupational switchers and stayers. Specifically, I divide the total number of workers who switch occupations by the sum of the occupational switchers and occupational stayers, weighted by sample weight. Note that I only measure the occupational mobility of employed workers.[16]

The baseline specification is the following two-way fixed effect model:

$$\left(\frac{\text{Switcher}}{\text{Stayer+Switcher}}\right)_{st} = \alpha + \beta lnMW_{st} + \delta_t + \lambda_s + \Gamma Y_{st} + \epsilon_{st} \tag{16}$$

I obtain log real minimum wages using the regional price index from BLS. δ_t is the month-year fixed effect. λ_s is the state fixed effect. Similar to the literature on the employment effect of minimum wages, I control for "supply factors" Y_{st} of occupational switching.

[14] The details are in section A.2.3.

[15] I use 4-digit 2002 Census code. Guvenen et al. (2018) also use this criterion for identifying occupational switch. I consider different measures in the appendix section A and the results are consistent.

[16] Kambourov and Manovskii (2010) suggest using a different measure of occupational switch. Specifically, they identify a switcher only if a worker experiences a change in her usual job activities. In this section, I do not use their measure for two reasons. The first reason is because of the large fraction of non-response: about 60% of the time there is no worker that meets the switcher criterion, averaged across time and states. This is unlikely an accurate reflection of the US labor market. The second reason is that Kambourov and Manovskii (2010) focus on the *level* of occupational switch, while I focus on the *changes* in occupational switching probability. As long as the measurement error does not vary systematically with minimum wages, it biases the estimates towards zero rather than exaggerating the estimates. In the appendix section A.1, I consider the measure in Kambourov and Manovskii (2010) using micro-level data and find similar results.

Specifically, I use the Quarterly Census of Employment and Wages (QCEW) to construct the share of manufacturing and retail employment for each state. If one thinks occupations in these industries have higher occupational mobility, and states with increasing employment share in these industries are more likely to increase their minimum wages, excluding such controls might bias the estimate upwards.

The model shows that the minimum wage affects low ability workers' occupational mobility more. I construct the dependent variable, occupational switching rate, for subgroups of workers. I first estimate the effect for workers of different ages. I define younger workers as those aged between 16 and 30. Workers aged between 30 and 45 are older workers. The average monthly occupational mobility rate for younger workers is 2.1% while it is 1.0% for older workers. I regress the two panel data of the occupational mobility rate using equation (16). The result in table 1 shows that minimum wages decrease younger workers' occupational mobility while there is no significant effect on older workers: a 10% minimum wage increase decreases younger workers' occupational mobility by 3%.[17] That is, if the minimum wage increases by 10% in a given month, out of 100 employed younger workers who were to switch occupations, 3 of them would now choose to stay instead.[18]

To further show the effect of the minimum wage on occupational mobility, I construct two more sets of occupational mobility rates, by education and by occupation. Specifically, I construct the state-level monthly occupational mobility rate for workers with high school degrees or less and workers with college degrees or more. When grouping by occupation, I construct occupational mobility rate for workers in the five lowest-average-wage occupations and the five highest-average-wage occupations. The results in table 1 suggest that minimum wages decrease high school workers' and low-wage-occupation workers' occupational mobility while not affecting other groups of workers. The magnitude is similar to the previous estimate: a 10% increase in the minimum wage decreases occupational mobility of the corresponding group by 3%.

I further restrict the sample to be high-school-educated workers less than 30. The result in table 1 is the same: a 10% increase in the minimum wage decreases younger, less-educated workers' monthly occupational mobility by 3%.

One disadvantage of the occupational switching measure is that it depends on employer switching. To rule out the possibility that the measure only captures job mobility rather than occupational mobility, I focus on workers who *only* switch employers. I con-

[17] It is calculated as $log(1.1) \times (-0.015)/2.1\%$.

[18] A back-of-the-envelope calculation using the estimates in Topel and Ward (1992) shows that employment-employment transitions accounts for about half of all the occupational switches. In the appendix section A.3, I use employment-unemployment-employment transitions to define occupational mobility and results are consistent.

struct the number of employer-only switchers by identifying them as workers who switch employers but *do not* switch occupations. If the previous results only measure employer switching or job mobility, we should expect the estimate to be negative. The result in table 1 suggests this is not the case: in all sub-group analysis the point estimates are highly insignificant and most of them are positive. In addition, the measure used in the micro-level analysis in the appendix section A.1 is independent of employer switching and the result is consistent. I conclude that the measure of occupational mobility is not merely about job mobility.[19]

When restricting occupational stayers to workers who do not switch employers, the results remain similar in magnitude and unchanged in significance. One can interpret the result as the minimum wage decreases occupational mobility by making a worker stay longer in her current job. The increase in tenure is a not sign of improved match quality because the matches already exists, unless one thinks the minimum wage changes certain aspects of the worker related to match quality.

I interpret the negative mobility response as a sign of reduced labor market dynamism. A related question is which occupational transitions are most affected by the minimum wage. I show in section A.4 that for younger, less-educated workers, the minimum wage significantly decreases the transition from non-routine manual occupations (e.g. food preparation, building cleaning) to routine cognitive occupations (e.g. office and administrative support). The result is more consistent with the minimum wage decreasing workers' search incentive rather than improving match quality. In the latter case, we should expect a decrease in the transition rates *within* non-routine manual or routine cognitive occupations.

4.1.1 Placebo Test and Adding State-Specific Time Trend

To partially address potential identification issues, I conduct a placebo test similar to Dube et al. (2010).[20] I separate the states into two similar sized subsets. The first subset of states is the infrequent changers: states that increase the minimum wage less often. The second subset is the frequent changers. The variation mainly comes from the states that have frequent minimum wage increases, and the controls are the states that do so less often. If

[19] This does not mean the minimum wage has *no effect* on job mobility, as recent evidence in Jardim et al. (2018) shows that minimum wages reduce job turnover rates.

[20] In Dube et al. (2010), they show that spacial correlation might make the estimate of employment effect of minimum wages less trustworthy by assigning the minimum wage policy of the states that have state-level minimum wages to states that only have federal minimum wages. If there is no spacial correlation, we should expect to see no effect in the placebo test.

these states are valid controls, assigning to them the minimum wage policy of frequent changers should not produce significant estimates.

I randomly assign the minimum wage policy of the frequent changers to the infrequent changers and run the regression equation (16) using only the infrequent changers.[21] Out of 500 permutations, only 6.4% of the estimates are significant, suggesting that the infrequent changers can serve as valid controls in the context of minimum wages on occupational mobility.

Another point mentioned in Dube et al. (2010) is that the results might not be robust to including state-specific time trends. If including state-specific time trends makes the estimate insignificant, some state-specific underlying trend might affect both the minimum wage policy and the outcome variable. I include in equation (16) state-specific time trends and the result shows that this inclusion barely changes the point estimate. The significance levels drop, but the change in p-value is small. Compared to the big changes in the estimates of the minimum wage on employment, the result is arguably robust to including state-specific time trends in the specific context.

4.1.2 Alternative Construction of Control Groups

As noted in Neumark et al. (2014) and Dube et al. (2010), estimating the effect of minimum wages hinges crucially on the choice of control groups. The regression equation (16) with additive time fixed effects is equivalent to subtracting off the mean value of the outcome and the explanatory variables for each time period, giving equal weight to each state for this de-meaning.

Several studies use alternative ways to construct control groups. For example, Dube et al. (2010) use geographical variation to construct control groups. I consider the Generalized Synthetic Control (GSC) developed in Powell (2016). The method extends the case study synthetic control method to more general settings and constructs the control groups by assigning weight optimally.

Using GSC, I estimate the effect of minimum wages on younger, less-educated workers' occupational mobility. The point estimate is -0.016, which is similar to the previous result in magnitude.

Unlike the classic synthetic control estimators, Powell (2016) does not specify methods to validate the GSC approach. I consider two methods based on the classic synthetic control literature. In the first method, I calculate the average correlation in the occupational mobility rates of each state and its generalized synthetic control after subtracting

[21] The details of the permutation algorithm is in the appendix section A.2.1.

off the effect of the minimum wage. I find that the average correlation increases from 0.5 to 0.75 when switching from the two-way fixed effect model to GSC, indicating that GSC produces better controls.

In the second method, I plot the occupational mobility rates for each state and its generalized synthetic control after subtracting off the effect of minimum wages. The GSC does not allow one to define "pre-trend". Rather, I show the overall fits between each state and its generalized synthetic control. The fits are good for most states.[22]

4.2 The Effect of the Minimum Wage on Mismatch

Besides a decrease in occupational mobility, the wage compression channel implies that a higher minimum wage leads to more mismatch in the stationary equilibrium. The workers search less and stay in mismatch longer after the minimum wage increase, leading to slower labor market dynamism and an increase in job tenure. To test the implication empirically, I use the data from Guvenen et al. (2018) to study the relation between minimum wage and mismatch. My model defines occupations and workers following their model. Importantly, they directly measure mismatch rather than using proxies such as wages. They measure occupation skill composition using O*NET and workers' ability to learn using NLSY79 ASVAB test scores. The Euclidean distance between skill compositions of occupations and ASVAB test scores defines mismatch. I augment their data with region-level minimum wages defined as the unweighted average minimum wages in a region.[23]

I conduct the analysis at the individual level, restricting the sample to white and Hispanic workers between 16 and 30 with a high school degree, controlling for age, race, education, gender, initial ability, year fixed effect and region fixed effect. The result shows that the minimum wage increases younger, less-educated workers' mismatch. I view the positive correlation as evidence that the minimum wage leads to more mismatch by the wage compression channel. The details are in the appendix section A.6.

5 Model Estimation and Quantitative Analysis

I estimate the model parameters in the steady state using the CPS data from 2008 to 2016 and the empirical results from section 4. I calculate the baseline job finding rate λ and

[22] The figures are in the appendix section D.

[23] The NLSY79 public data only has region information. There are four regions: Northeast, North Central, South, West. I use region-level price index to calculate the real minimum wage.

the separation rate δ as in Shimer (2005). The results are 0.36 and 0.02. I set the minimum wage to be \$7.25 per hour, equal to the federal minimum wage. I use states with minimum wages equal to \$7.25 at the end of 2016 to construct the moment targets.[24]

The ability parameter $\boldsymbol{a}$ is fixed to be one-dimensional corresponding to education. Similarly, I set $\boldsymbol{j}$ to be one-dimensional corresponding to occupation skill intensity, as in Autor and Dorn (2013).

I discretize the ability and the occupation skill intensity intervals into ten grid points. Each ability grid point has one point in the occupation grid that corresponds to its optimal occupation. The distribution $Beta(\kappa_1, \kappa_2)$ determines the measure of workers of each ability grid point.

I calibrate (κ_1, κ_2) to match the workers' education-level distribution in the CPS. Specifically, the workers are divided into three groups by education achievement: 1. high school degree or less. 2. some college or associate degree. 3. bachelor's degree and above. In the sample, 29.1% of the workers complete college degree, 28.6% of the workers have an associate degree or vocational training, and 42.3% of the workers have a high school degree or less. I categorize grid 1 and 4 to be the low ability workers, 5 to 7 to be the medium ability workers, and 8 to 10 to be the high ability workers. The parameters of the Beta distribution is calibrated to match the empirical composition of the workers by education. The resulting distribution is $Beta(0.8879, 0.9414)$.

The accuracy of finding good matches affects workers' occupational mobility. The workers switch less if they are on average in good matches. If every worker can target their optimal occupation with certainty, there will be no occupational mobility because sorting is perfect. I introduce a parameter ρ that governs the probability that the workers will end up in their optimal occupations when searching on the job or transit from unemployment to employment. A worker has probability ρ of finding her optimal occupation and equal probability of finding any of the other occupations, given by $(1-\rho)/9$. If ρ is 0.1, it is equivalent to the workers not targeting any occupation. ρ and the workers' ability distribution implicitly determine the joint distribution of occupations and ability.

ρ also affects the gain from switching. If $\rho > 0.1$ and there is no switching cost, occupational switching is good because workers have a higher probability of moving to their optimal occupations. To calibrate the parameter ρ, I target the average percentage wage gain after switching occupations. In the model, the wage change comes from changes in mismatch because productivity is match-specific. This means that the moment target cannot come from comparing wages before and after an occupational switch in the data. The wage changes in the data overestimate the wage gain from switching occupations by

[24] A list of the federal minimum wage states is in table A.10.

incorporating the gain from tenure.

I use the estimates in Guvenen et al. (2018) in which a back-of-the-envelope calculation shows that on average workers experience 1% wage growth attributed to a reduction in mismatch when switching occupations.[25]

Another key determinant of occupational mobility is the on-the-job-search threshold x_s. It is a function of a: the ability directly affects the search threshold, differentiating the search thresholds of high ability workers and low ability workers. It is also a function of the minimum wage, as shown in section 3. I expand the function as

$$x_s(a,m) = s_0 + s_1 a + s_2 \mathbb{I}_{(a<qm)} m \tag{17}$$

The indicator function allows me to compactly capture the non-linear effect of minimum wages on occupational mobility with one parameter q. The constant s_0 absorbs the cost of switching. Twice continuous differentiability of the value functions justify the Taylor expansion.

The endogenous separation cutoff directly relates to the *employment* effect of the minimum wage. I estimate jointly the endogenous separation cutoff and the vacancy rate or equivalently the job finding rate to target unemployment rate by education and the employment elasticity of the minimum wage by education. Specifically, I use the following linear Taylor expansion for estimating the endogenous separation cutoff:

$$\underline{x}(a,m) = p_0 + p_1 a + p_2 m \tag{18}$$

I use λ' to denote the job finding rate after the minimum wage increase.

I discretize the output process by the Euler-Maruyama approximation:

$$X_{t+1} = X_t + \tilde{a} X_t \Delta_t + \sigma X_t \mathbb{N} \sqrt{\Delta_t}$$

$\mathbb{N}$ is a standard normal random variable. The choice of time step $\Delta_t = 0.01$ allows the approximation to track the exact solution of the output SDE closely. I set the functional form of $\tilde{a}$ to be $\tilde{a}(a, |a-j|) = a/(1+|a-j|)$, in which $\tilde{a}$ is increasing in ability a and decreasing in mismatch $|a-j|$.

The other moments I target include the occupational mobility rates by education, the wage distribution statistics, and the effect of the minimum wage on occupational mobility.

In the sample, the monthly average occupational mobility rate is 2.6% for high school

[25] The addition of the parameter ρ does not change any of the result in section 2 and section 3. The implied joint CDF $N(a,j)$ would have more weight along the trace if $\rho > 0.1$.

workers, 1.5% for associate degree workers, and 1.1% for bachelor degree workers. I set them as targets for low ability, medium ability, and high ability workers in the model. Similarly, the unemployment rate target is 7.5% for the low ability workers, 6.2% for the medium ability workers, and 3.6% for the high ability workers.

I target the empirical estimates on the effect of minimum wages on occupational mobility. By section 4, a 10% minimum wage decreases younger, less-educated workers' occupational mobility by 3%. The model targets the occupational mobility elasticity of the minimum wage to be 3% for the low ability workers. There is no empirical evidence that the minimum wage affects the medium and high ability workers. I set the elasticity targets to be 0 for both types.

The model also targets the employment elasticity of the minimum wage. The targets depend on workers' types. Despite the different findings in the literature, it is arguably the case that the employment effect on workers with at least some associate degree is small. I set the targets to be 0 for the medium and the high ability workers. For the low ability workers, I set the elasticity to be -0.1 which is on the smaller end of the spectrum in magnitude among the literature that finds negative employment effect of minimum wages.

The moment targets of the employment elasticity of the minimum wage imply that the vacancy posting and the job arrival rate cannot change too much even after a large minimum wage increase. I minimize the employment channel and focus on the wage compression channel, which is independent of the employment effect channel.

I include several wage distribution statistics. The first set of statistics is the wage distribution ratios. Specifically, I include the P50/P10, P40/P10, P30/P10, P20/P10 ratios, in which P10 stands for the 10th percentile in the wage distribution. The second set of moments relate to the extent to which the minimum wage binds. Using the CPS data, I find that 5.1% of the workers earn binding minimum wages during the sample period, and 40% of the workers face a binding minimum wage if the minimum wage increases to $15 in 2020. These estimates are consistent with the report by the National Employment Law Project.[26] I also include the mean to variance ratio of the wage distribution as a target because it relates to σ in equation (1).

Together there are 10 parameters and 20 moments.[27] I use the GMM method to estimate the parameters following Lise and Robin (2017). Specifically, I simulate the model with a $7.25 minimum wage until it reaches the steady state, and then increase the minimum

[26]Source: https://www.nelp.org/publication/growing-movement-15/. I project a 2% annual wage growth into 2020 and impose a $15 minimum wage to calculate the fraction of workers facing a binding minimum wage.

[27]Details of the moments value is in the appendix section C.

wage by 10% and compare the employment and occupational mobility between the two steady states, which gives the elasticity of employment and occupational mobility with respect to the minimum wage. I calculate the other steady state moment targets under the $7.25 minimum wage. Table 2 shows the corresponding parameter values.

Eight out of ten parameters are significant at the 5% level. The change in vacancy posting, or equivalently the change in job arrival rate λ' is small: a 100%% increase in minimum wage only decreases job arrival rate by 1.4%. This is because the job arrival rate affects the occupational mobility and employment of all workers equally, regardless of the ability. It needs to be small, so that there is no effect of minimum wages on occupational mobility and employment for the medium and the high ability workers.[28] Figure 3 plots the empirical wage distribution in blue and the simulated wage distribution in orange. The simulated wage distribution gives more weight on the medium wage range and falls short of accounting for the density in low wage and extreme high wage range. The overall shape and the decay of the right tail match the empirical distribution.[29]

I plot the average wage by workers' ability in (a) of figure 4. The average wage increases as the worker's ability goes up. The sharp rise shows that ability difference contributes to the fat right tail of the wage distribution. The increase in average wage by occupation skill intensity is less significant, as shown in (b) of figure 4. The comparison implies that sorting is imperfect because workers cannot target their optimal occupations with certainty.

5.1 The Effect of Minimum Wage on Occupational Mobility

The empirical analysis in section 4 shows that recent small minimum wage changes decrease the occupational mobility of younger, less-educated workers moderately. To study the effect of large minimum wage changes, specifically a $15 minimum wage on occupational mobility, I increase the minimum wage from $7.25 to $15 in the model. The increase is about 100% and affects 40% of the labor force. I simulate the model for a 1000 times and calculate the average decrease in occupational mobility.[30] Because the effect of the minimum wage depends on the wage distribution, the experiment is specific to the case where the current minimum wage is $7.25, a 10% increase in the minimum wage induces the targeted elasticity, and a $15 minimum wage binds for 40% of the labor force.

The large minimum wage increase has differential effects on workers' occupational

[28] A direct estimate of the effect of minimum wages on vacancy posting would be desirable but is unavailable due to data limitation.

[29] Table C.1 in the appendix section C lists the moment targets data and the model estimates.

[30] A $15 nation-wide minimum wage is one of the Democratic party's platform in the 2016 election. It is also currently under discussion. See e.g. Clemens (2019).

mobility. The results show that the increase in the minimum wage dis-incentivizes the lower ability workers the most: the 100% increase in the minimum wage decreases their occupational mobility by as much as 44%. The medium and high ability workers do not experience any significant decrease in their occupational mobility. Overall, occupational mobility falls by 30%.

The decrease in occupational mobility is a combination of the employment effect and the wage compression effect. I restrict the model to have only the wage compression effect and the model estimate matches the linear extrapolation of the empirical result, as shown in (a) of figure 5. Adding the employment effect further decreases occupational mobility. The effect is small compared to the wage compression channel. The wage compression channel accounts for 90% of the decrease in occupational mobility.

As the percentage increase in the minimum wage becomes large, the slope of the model estimate becomes steeper. If I allow for a continuum of worker types and higher order terms in equation (17), the model estimate in figure 5 would be a concave, non-linear, and smooth curve. The non-linearity comes from the shape of the empirical wage distribution. When the minimum wage is at \$7.25, the wage distribution is flat. A small increase in the minimum wage affects a small fraction of workers. Further minimum wage increase cuts into the part of the wage distribution where it rises steeply, which non-linearly increases the fraction of workers facing a binding minimum wage.

Figure 6 (a) illustrates the fraction of workers with the binding minimum wage across occupations. Two features stand out in figure 6 (a). First, the fraction of workers with binding minimum wages differs significantly across occupations: as we move from low-average-wage occupations to high-average-wage occupations, the fraction of workers facing a binding minimum wage declines. Second, when the minimum wage increases to \$15, some occupations more than triple the fraction of workers facing the binding minimum wage, which replicates the findings in figure 7. There is also a non-trivial fraction of workers earning the minimum wage in high paying occupations, reflecting that the model generates substantial wage inequality *within* occupations. Figure 8 shows the large wage inequality within occupations. The wage inequality comes from mismatched workers. Within each occupation, there are both low and high ability workers. It also comes from stochastic output.

Figure 7 shows the empirical counterpart of figure 6 (a). It plots the fraction of workers with a binding minimum wage across 2-digit occupations, ranked by average wage. I project the wage into 2020 at a 2% annual growth rate and impose a \$15 minimum wage. The similarity in figure 6 and figure 7 shows that the \$15 minimum wage significantly increases the fraction of workers with a binding minimum wage. I also plot the fraction of workers with a binding minimum wage across ability in (b) of figure 6. At \$7.25, only low ability workers earn the minimum wage, while the \$15 minimum wage binds even for a large fraction of medium ability workers.

5.2 Occupational Mobility Response and Aggregate Output

What is the implication of the negative mobility response? Both the employment effect channel and the wage compression channel cause the negative mobility response. Less employment would damp aggregate output. On the other-hand, for workers who remain employed, negative mobility response could also decrease aggregate output because workers stay in mismatch longer. A higher minimum wage slows down on-the-job occupational switching towards better matches. The slow-down would lead to an increase in aggregate mismatch and hence a decrease in output.

To understand the extent to which the minimum wage increases aggregate mismatch, I compare mismatch before and after the minimum wage increase. I increase the minimum wage to \$15 and simulate the model for a 1000 times. For an employed worker with ability a, I calculate $\mathbb{I}_{(|a-j|>0)}$ which is the probability that she is not in her optimal occupation. I average the probability across all workers and all periods. The number captures the average time spent in non-optimal occupations. The \$15 minimum wage makes the low ability workers 2% more likely to be in non-optimal occupations. For the medium and the high ability workers, the changes in the probability are insignificant.

The increase in mismatch translates to a decrease in aggregate output. On average, low ability workers' output decreases significantly by 1.3%. The output of the medium and the high ability workers are not affected. Overall, the \$15 minimum wage decreases aggregate output by as much as 0.4%. The 0.4% decrease in aggregate output is large. This is because the fraction of workers influenced by the \$15 minimum wage is 40%. Of the 0.4% decrease in aggregate output, the wage compression channel accounts for 80%, or 0.0032 percentage points.[31]

The result suggests that a large minimum wage increase should have different short-run and long-run effects. In the short run, the minimum wage increase does not affect occupational mobility. The aggregate output distribution remains unchanged. In the long run, low ability workers decrease their occupational switching in the new equilibrium. Compared to the output distribution before the minimum wage increase, a larger fraction of low ability workers now remain in mismatched occupations. The effect is a leftward shift of the density of the output distribution. The minimum wage affects the whole wage

[31] Details of the decomposition can be found in the appendix section C.

distribution by changing workers' search behavior than simply truncating the wage distribution.

The above results have important policy implications. The discussion of the minimum wage policy focuses on its employment effect, but the evidence in this paper shows that the minimum wage decreases occupational mobility by the wage compression channel. The wage compression is independent of the employment effect channel. The reduced incentive of on-the-job occupational switching slows down workers' improvement in match quality, which leads to more mismatch and hence less aggregate output.

6 Conclusion

This paper identifies a novel workers' response to the minimum wage, namely occupational mobility. I study the effect of the minimum wage on occupational mobility and the implication on aggregate output. I construct a search-and-matching model and highlight two channels by which the minimum wage decreases occupational mobility. The first channel is the employment effect in which the minimum wage displaces workers as well as decreases firms' vacancy posting and hence job arrival rate. The second channel is the wage compression channel in that the minimum wage reduces the gain of switching to better matched occupations. The negative mobility response after the minimum wage increase induces more mismatch. I document empirically that the minimum wage is negatively correlated with the occupational mobility of younger, less-educated workers and positively correlated with mismatch, supporting the model's implications.

I estimate the model to study the effect of a large minimum wage increase. In the estimated model, a $15 minimum wage decreases low ability workers' occupational mobility by 44%, leading to an increase in mismatch concentrated on the low ability workers. Because mismatch reduces output, the negative mobility response shifts more weight of the output distribution to the left tail. The decrease in output is 0.4%, of which the wage compression channel accounts for 80%. The results have important policy implications: even if employment does not decrease, a large minimum wage increase can still lead to a non-trivial decrease in aggregate output by the wage compression channel which leads to slower labor market dynamism and more mismatch.

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Tables and Figures

Table 1: The Effect of Minimum Wages on Occupational Mobility

	(1) Age 16-30	(2) Age 30-45	(3) High School	(4) College	(5) 5 Lowest-Wage Occupations	(6) 5 Highest-Wage Occupations	(7) Age 16-30 × High School	(8) GSC
					Using Employer Switching Measure			
$lnMW_t$	-0.015***	-0.0015	-0.0077**	0.0014	-0.009***	0.001	-0.019**	-0.016*
	(0.0040)	(0.0031)	(0.0038)	(0.0035)	(0.0031)	(0.0041)	(0.0071)	p = 0.091
					With State-Specific Time Trends			
$lnMW_t$	-0.013***	-0.0007	-0.0061	0.0020	-0.011***	0.0001	-0.016**	—
	(0.0046)	(0.0030)	(0.0045)	(0.0030)	(0.0037)	(0.005)	(0.0080)	
					Employer Switching without Occupational Switching			
$lnMW_t$	0.0125	—	0.0119	—	0.0327	—	-0.0105	—
	(0.0215)		(0.0098)		(0.0226)		(0.0095)	
N	5508	5508	5508	5508	5508	5508	5508	5508
State FE	Y	Y	Y	Y	Y	Y	Y	
Year Month FE	Y	Y	Y	Y	Y	Y	Y	

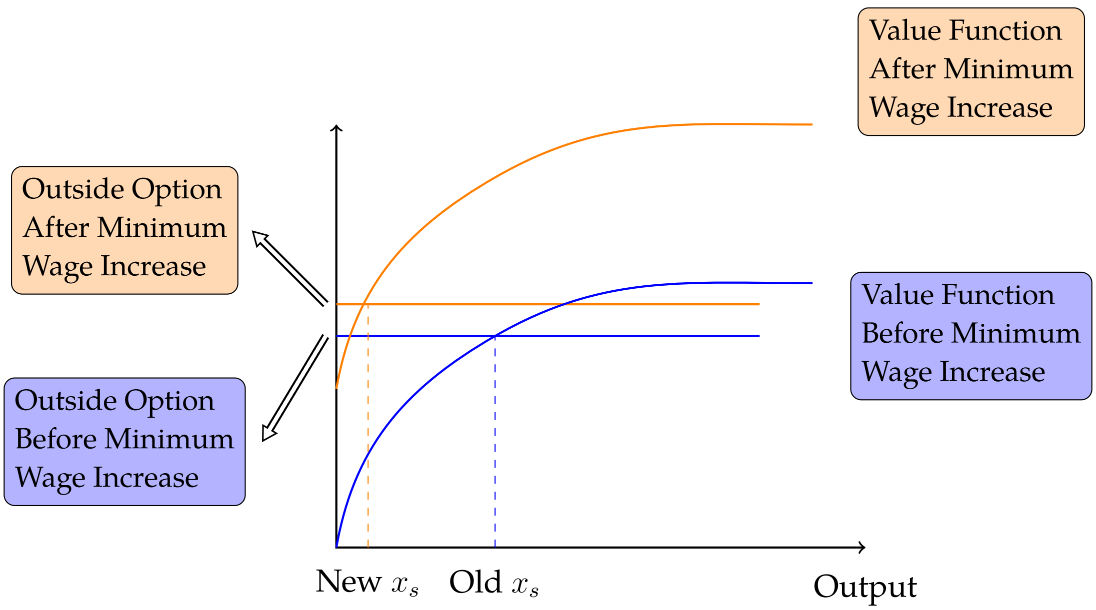
Notes. The sample period is from 2008 to 2016. The columns present the results on the occupational mobility of corresponding sub-groups. The first row of regression uses state-level occupational mobility at the monthly frequency as the dependent variable. An occupational code change counts as a switch only if there is accompanying employer switching. The second row of regression adds state-specific time trends. The third row uses the number of workers who switch employers without changing occupations as the dependent variable, normalized by dividing by 10,000. All three sets of regressions include the share of manufacture and retail trade employment as controls. The state-clustered standard error for the two-way fixed effect model is in parenthesis. *** means statistically different from zero at the 1% level, ** at the 5% level, * at 10% the level. The inference for the GSC uses a Wald statistic. For details, see Powell (2016).

Table 2: Parameter Estimation Results

Parameters			
ρ	0.498**	σ	0.72**
	(0.235)		(0.262)
s_0	1.05**	p_0	0.65
	(0.031)		(1.347)
s_1	-0.2**	p_1	-0.55
	(0.041)		(0.363)
s_2	-0.009**	p_2	0.008**
	(0.001)		(0.001)
λ'	0.355**	q	0.028**
	(0.006)		(0.001)

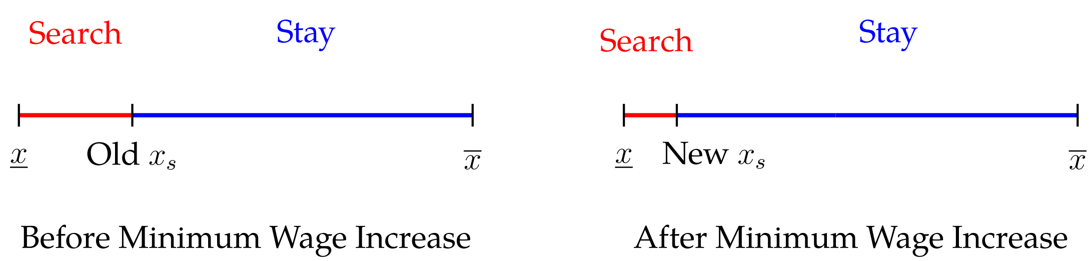
Notes. ** means statistically different from zero at the 5% level. ρ dictates how accurately workers can target their optimal occupations. $\{s\}_{(0,1,2)}$ are the Taylor expansion coefficients for the on-the-job search cutoff (Equation (7)). $\{p\}_{(0,1,2)}$ are the Taylor expansion coefficients for the endogenous separation cutoff. λ' is the stationary equilibrium job arrival rate after a 10% increase in the minimum wage from \$7.25. σ is the diffusion coefficient of the productivity process. q captures the non-linearity in the on-the-job search cutoff.

Figure 1: The Wage Compression Channel



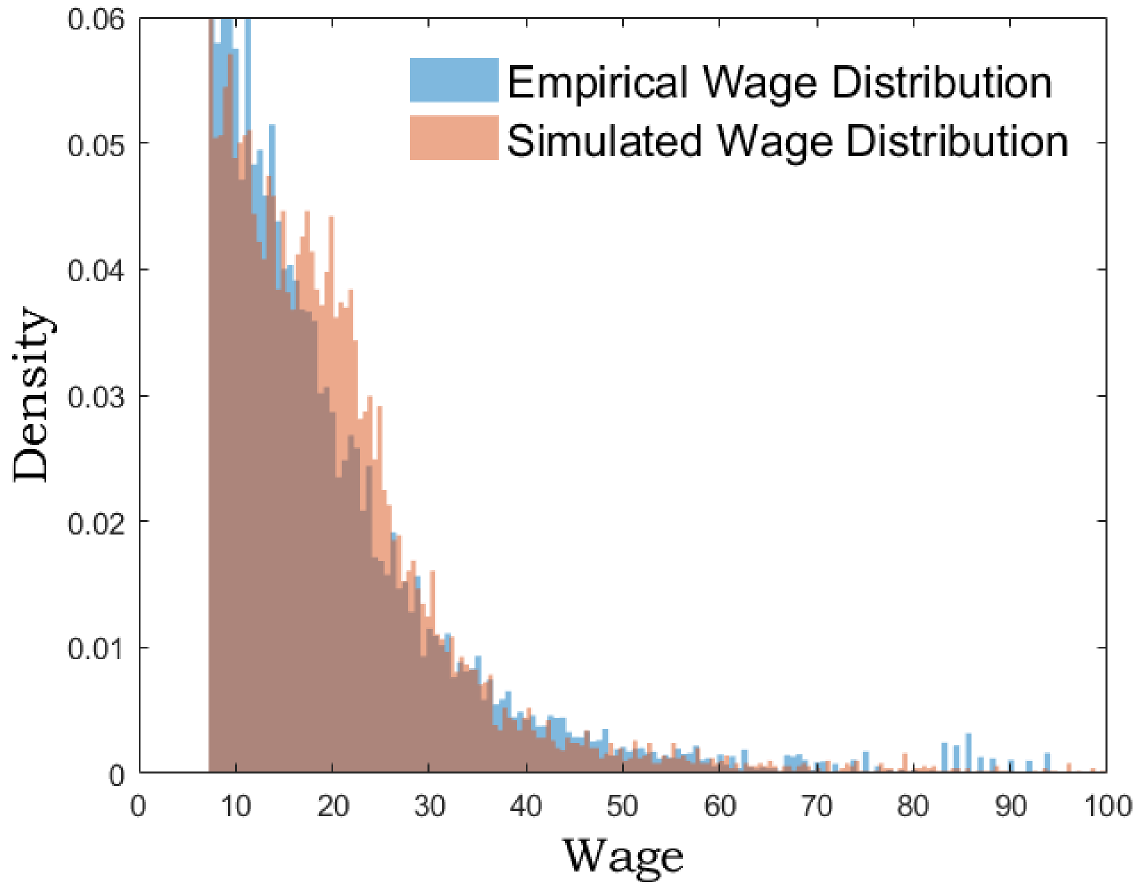
Notes. Figure 1 shows how the wage compression channel works. The blue curve is the value function before the minimum wage increase. The blue horizontal line is the outside option on the job. The point they intersect—old x_s—is the on-the-job search cutoff point. The worker stays in the current occupation if the output exceeds old x_s, and searches if the output is below it. The orange curve is the value function after the minimum wage increase. The upward shift is substantial because the worker is in a mismatch, so that the minimum wage is more binding for her. The worker is better matched in other occupations, so that the minimum wage affects the value of the outside option less. This causes the intersect of the two curves to move to the left to new x_s.

Figure 2: On-the-Job Search and Occupational Mobility



Notes. Figure 2 illustrates the effect of a leftward shift in on-the-job search cutoff on occupational mobility. Notably, the search region shrinks. Workers with output between new x_s and odl x_s stop searching on the job after the minimum wage increase, leading to a lower occupational mobility.

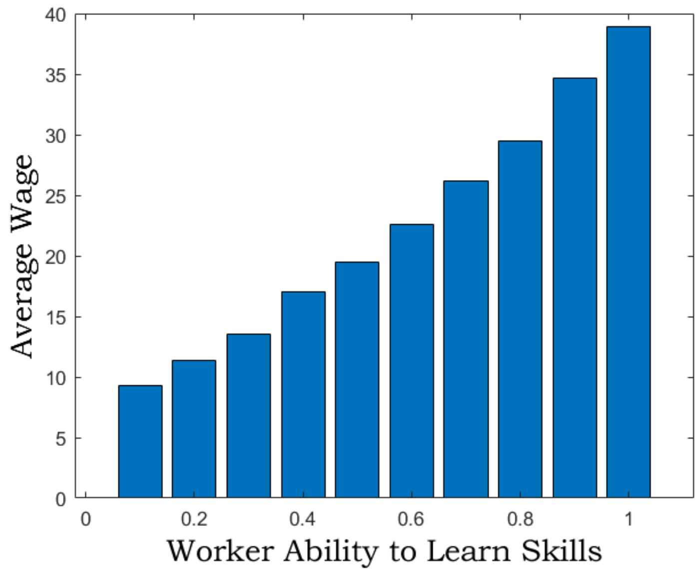
Figure 3: Model Simulated and Empirical Wage Distribution



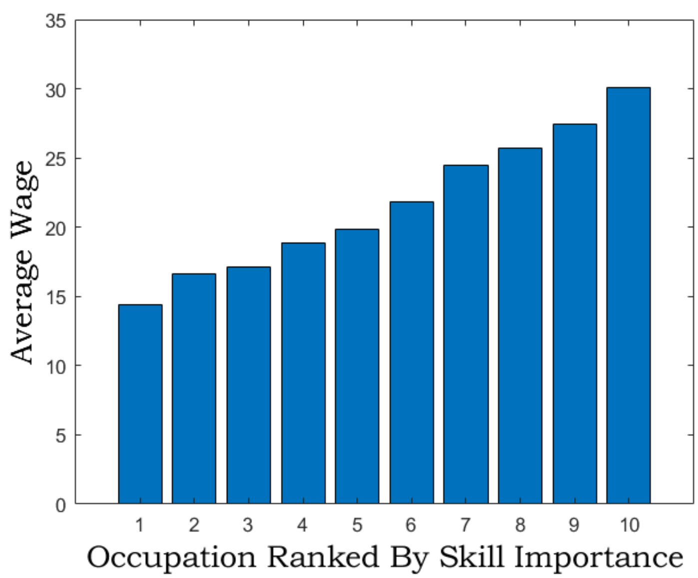
Notes. Figure 3 plots the empirical wage distribution in blue and the model simulated wage distribution in orange. Figure 3 estimates the empirical wage distribution using CPS merged outgoing rotation group data from 2008 to 2016. I calculate the real wages using the 2012 chain-type price index. The simulated wage distribution comes from averaging 500 periods of the realized wages in the estimated model when the minimum wage is $7.25.

Figure 4: Model Simulated Average Wage

(a) Average Wage by Ability



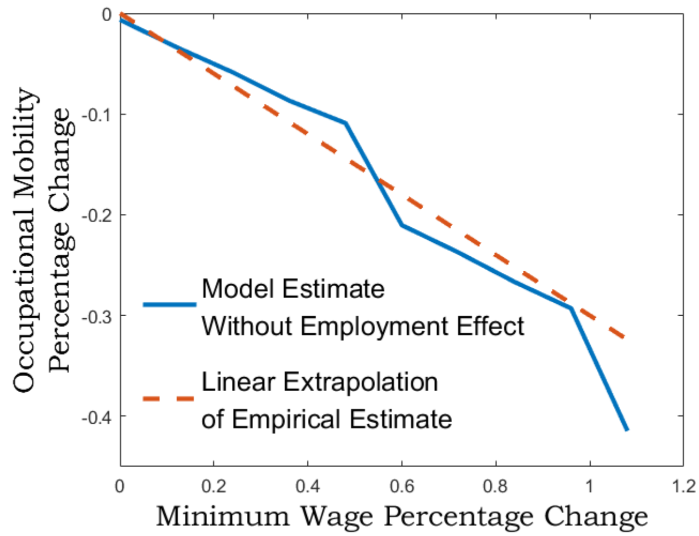
(b) Average Wage by Occupation



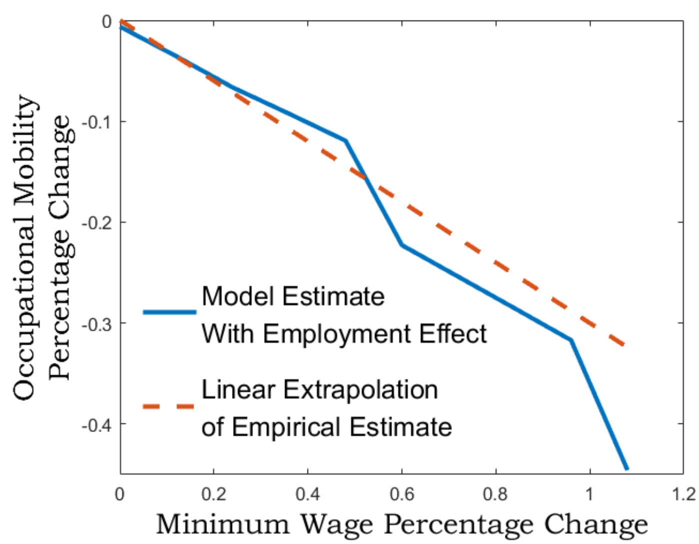
Notes. Figure 4 plots the model simulated average wage by worker's ability in (a) and by occupation skill importance in (b). The average wage increases more rapidly by ability, and less so by occupation. The comparison indicates that sorting is imperfect and there are mismatches in the equilibrium. Otherwise the two figures would look identical.

Figure 5: The Effect of Minimum Wages on Occupational Mobility

(a) No Employment Effect



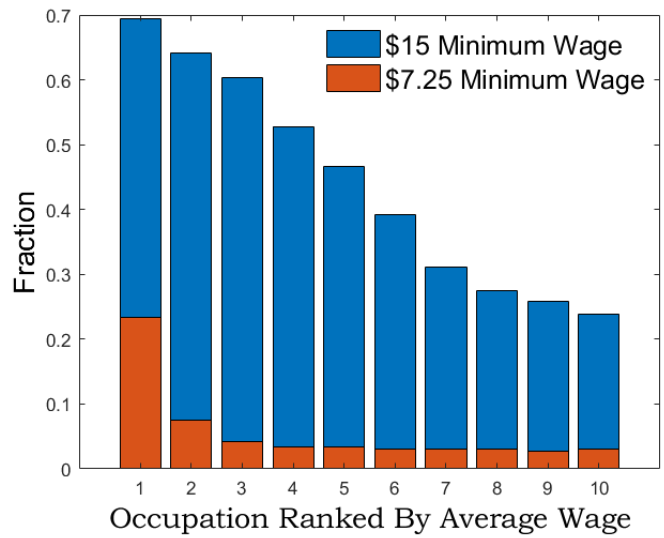
(b) With Employment Effect



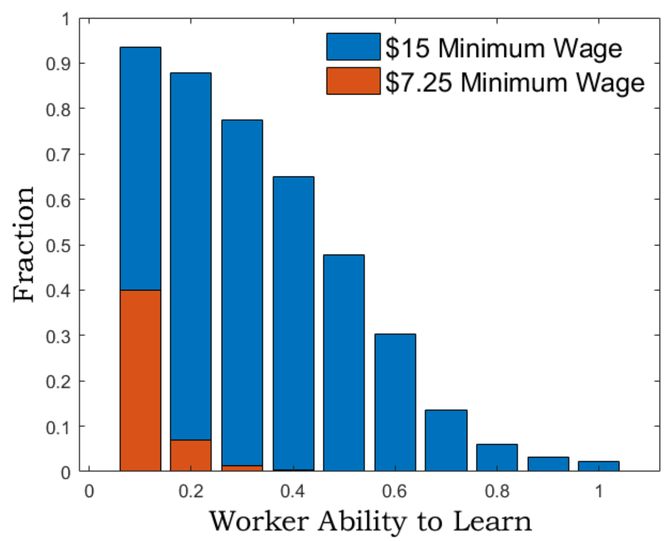
Notes. Figure 5 plots the percentage decrease in occupational mobility as a function of the percentage increase in the minimum wage. In figure (a) I shut down the employment effect channel by holding the job arrive rate constant and making the disemployment effect to be 0. The dash line is the percentage decline in occupational mobility based on the linear extrapolation of the empirical estimate in section 4. In figure (b) I plot the overall change in occupational mobility, which looks similar to (a). The comparison suggests that the wage compression channel accounts for a large fraction of the decrease.

Figure 6: Model Fraction of Workers with a Binding Minimum Wage

(a) Fraction of Workers with a Binding Minimum Wage by Occupations

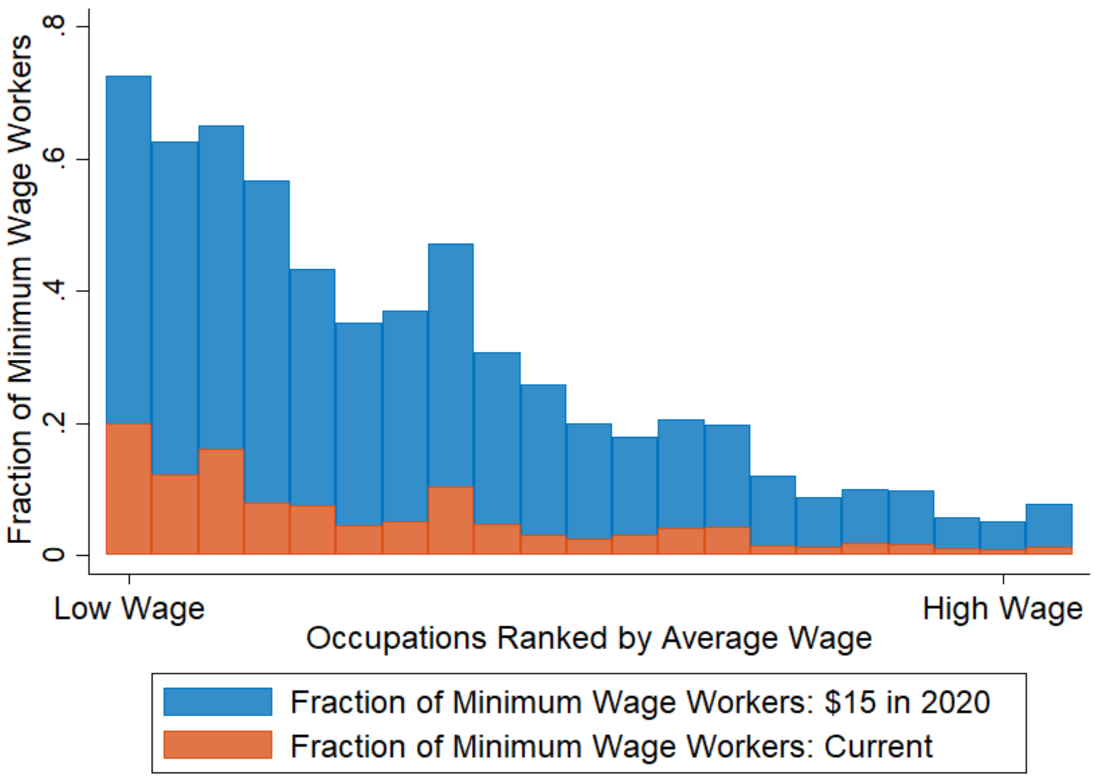


(b) Fraction of Workers with a Binding Minimum Wage by Ability



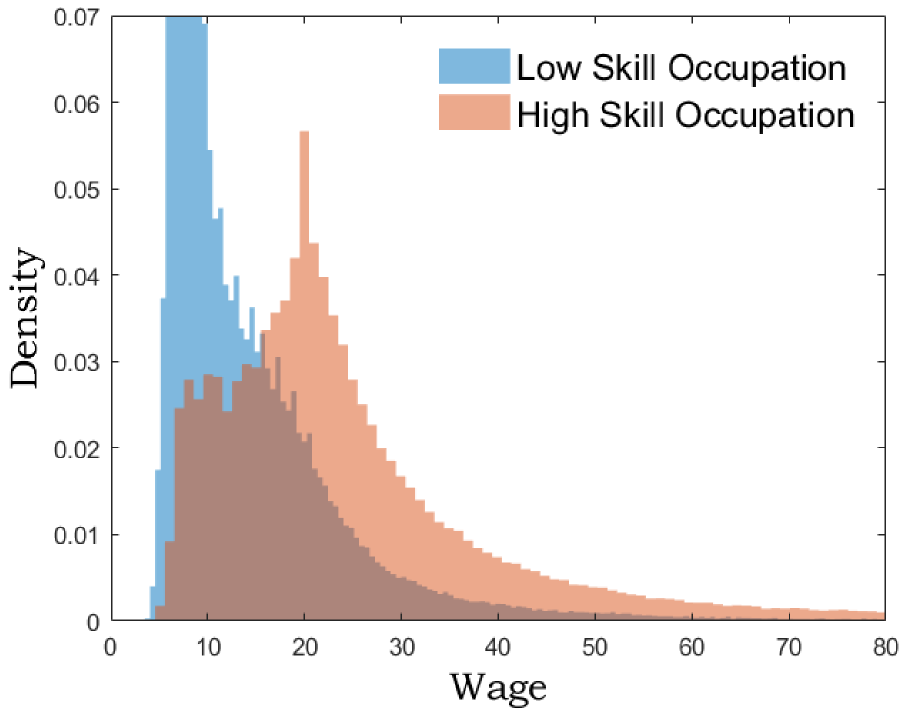
Notes. Figure 6 plots the fraction of workers with a binding minimum wage by occupation ranked by average wage in (a) and by workers' ability in (b). Figure (a) is similar to figure 7, indicating the model matches the data well. When the minimum wage is $7.25, few workers earn the minimum wage in high-wage occupations. When the minimum wage increases to $15, the fraction of workers with a binding minimum wage increases substantially in all occupations. In figure (b), it shows that a large fraction of medium ability workers earn the $15 minimum wage, while only low ability workers earn the $7.25 minimum wage. The comparison demonstrates the non-linear effect of the minimum wage on the fraction of minimum wage workers.

Figure 7: Fraction of Minimum Wage Workers by Occupations



Notes. Figure 7 plots the fraction of workers with binding minimum wages across occupations. The estimates is from CPS merged outgoing rotation groups from 2008 to 2016. Occupation code uses 2-digit 2002 Census code. There are 22 occupations. I rank them by average wage and plot them on the x-axis. The red bar plot shows the fraction of workers with binding minimum wages under current minimum wages. I project the wages to grow at 2% a year into 2020 to calculate the estimated fraction of workers with a binding minimum wage assuming a $15 minimum wage in 2020.

Figure 8: Wage Distribution within Occupations



Notes. Figure 8 plots the wage distributions for the low skill occupation (grid 1) in blue and the high skill occupation (grid 10) in orange. There are substantial wage dispersion within occupations. This is in part because of stochastic output. Mismatch also drives the dispersion.

Appendices

A Data Construction and Robustness Check

A.1 Micro-level Data Analysis

I first discuss my measures of occupational switching in the subsection. I utilize the dependent coding system in CPS and seasonally adjust the data using the ratio-to-moving-average method as in Shimer (2012). State and federal level minimum wage data comes from Vaghul and Zipperer (2016). I use the regional price index to calculate real minimum wages.

The CPS dependent coding system provides extra information on labor market mobility. Besides specifying the occupational code of each individual, the CPS asks the following questions:

1. Last month, it was reported that you worked for (employer's name). Do you still work for (employer's name) (at your main job)?

2. Have the usual activities and duties of your job changed since last month?

3. Last month you were reported as (a/an) (occupation) and your usual activities were (description). Is this an accurate description of your current job?

In section 4 I use the first question to identify occupational switching. Guvenen et al. (2018) also use the measure. The advantage of using the measure is that it significantly reduces fictional occupational switches because of mis-coding of occupations and it suffers less from non-response compared to using question 2 or 3. The average monthly occupational mobility of young workers, using employer switching measure, is 2.1%, which is consistent with Kambourov and Manovskii (2010). Without using employer switching, the number jumps up to 12%, which is unlikely. The disadvantage is that the measure depends on employer switching.

In this section, I also use question 3 to measure occupational switching. Using question 3, according to Kambourov and Manovskii (2010), produces the most accurate account of occupational switching. The main motivation in Kambourov and Manovskii (2010) for using this measure is that the level of occupational mobility is more accurate. Because I focus on changes in occupational mobility, the level accuracy is less relevant. For my purpose, the main advantage comes from its independence of employer switch. The disadvantage is a large fraction of non-response. When I aggregate the occupational mobility to the state-monthly level using the measure, around 70% of the observations are empty. This means I can only use micro-level data for the measure.

I use micro-level data for the measure to study the effect of the minimum wage on occupational mobility. My regression specification is:

$$\text{Occupational Switch}_{ist} = \alpha + \beta lnMW_{st} + \delta_t + \lambda_s + \tau_s * t + \Gamma X_i + \Omega Y_{st} + \epsilon_{ist} \tag{A.1}$$

I include state-specific time trends to address the potential confounding factor, pointed out in Dube et al. (2010). The sample includes teenagers (aged 16 to 19) in low-wage occupations with a high school degree or less. My sample period is 2008 to 2016. Section A.5 gives a detail list of the low wage occupations. The regression is weighted by sample weight.

Table A.1 shows the results. The baseline regression with no state-specific time trends shows that the minimum wage decreases occupational mobility under both measures. The result in the first column uses question 1 to define occupational mobility, and the result in the second column uses question 3. Both estimates are negative and significant. Because the measure using question 3 is independent of employer switching, I argue that the minimum wage decreases teenage workers' occupational mobility beyond job mobility. Including state-specific time trends reduces the significance of the estimate.

A.2 Robustness

A.2.1 Placebo Test Details

I illustrate the placebo test in section 4 in detail. I first calculate the number of minimum wage increases during the sample period for each state. It turns out that Iowa has the lowest number of 0, while New York has the highest number of 10 times. 33 states change minimum wage less than five times, while the other 18 states have five or more minimum wage increases. I refer to the first group as the infrequent changers and the second group as the frequent changers. The infrequent changers is similar to the placebo sample in Dube et al. (2010) and the frequent changers is similar to the actual sample in Dube et al. (2010).

I randomly assign the minimum wage policies of the frequent changers to the infrequent changers in a one-to-one fashion. I first permute the minimum wage policies among the frequent changers, then assign them to 18 randomly chosen states from the infrequent changers. The total number of mappings is $18! \times C(33, 18)$ in which C denotes the combination function. I take 500 of them and run regression equation (A.1) using only the infrequent changers, with the fictitious minimum wage data. Similar to the logic in Dube

Table A.1: The Effect of the Minimum Wage on Occupational Mobility

	(1) Employer Switch	(2) Usual Activity Change
	No State-Specific Time Trends	
$lnMW_t$	-0.038***	-0.014**
	(0.013)	(0.007)
	With State-Specific Time Trends	
$lnMW_t$	-0.021	-0.014*
	(0.016)	(0.008)
State FE	Y	Y
Time FE	Y	Y
Observations	N = 59632	N = 58917

Notes. The sample period is from 2008 to 2016. The first column measures occupational switching using employer switcher defined in appendix section A.1 question 1. The second column uses usual activity change defined in appendix section A.1 question 3. The controls include education, age, race, sex, state-level manufacture employment share and retail trade employment share. I weight the regression by three-month average final weight. I use state-clustered standard errors. *** means significant at 1% level, ** means significant at 5% level, * means significant at 10% level.

et al. (2010), if the infrequent changers are valid controls in the two-way fixed effect regression, we should not see any significant effect. Out of 500 repetitions, only 6.4% give significant estimates at the 5% level.

I repeat this exercise, but use the aggregate state-level occupational mobility of younger, less-educated workers as the dependent variable and run the regression equation (16). Out of the 500 estimates, only 5.6% of them are significant at the 5% level. This suggests that the infrequent changers are valid control groups in this context.

A.2.2 Endogenous Control Variables

Another potential issue not discussed in section 4 is that the control variables might be endogenous. If the minimum wage changes manufacturing and retail employment share, the effect would bias the estimates in the two-way fixed effect model. To show that this is not the case, I regress the controls on the minimum wage using the two-way fixed effect model. The result in table A.2 shows that the minimum wage has no significant effect on either of the controls.

Table A.2: The Effect of the Minimum Wage on Controls

	(1) Manufacturing Employment	(2) Retail Employment
$lmMW_t$	0.0009	0.0014
	(0.005)	(0.004)
Observations	5508	5508
R-squared	0.9889	0.9867
State FE	Y	Y
Year Month FE	Y	Y

Notes. The first column regresses state monthly manufacturing employment share on log real minimum wages and state and year-month fixed effects. The second column uses state monthly retail trade employment share as the dependent variable. The sample period is from 2008 to 2016. Table A.2 uses state-clustered standard errors.

A.2.3 Different Sample Periods and Effect on Job Finding Rate

I present regression results using equation (16) but with different sample periods. Table A.4 shows that the conclusion does not dependent on the specific choice of sample periods: negative response remains mostly consistent for young, less-educated workers or workers in low-wage occupations. The regressions do not include state-specific time trends.

I also construct job finding rate for young workers as in Shimer (2005). Using various sample periods, the results in table A.3 suggest that the minimum wage has no significant effect on young workers' job finding rate, except for the ones using data from 2012 to 2016. Interestingly, the minimum wage has no effect on their separation rate, consistent with the small estimates of the employment elasticity in the literature.

Table A.3: The Effect of Minimum Wages on Job Finding Rate and Separation Rate

	Job Finding Rate		Separation Rate	
	(1) Age 16-30	(2) Age 16-30 × High School	(3) Age 16-30	(4) Age 16-30 × High School
		2004 to 2016 (N = 7922)		
$lnMW_t$	-0.0360	-0.0518	0.001	0.001
	(0.0461)	(0.0488)	(0.006)	(0.006)
		2006 to 2016 (N = 6698)		
$lnMW_t$	-0.0549	-0.0762	0.001	0.001
	(0.0462)	(0.0500)	(0.006)	(0.006)
		2008 to 2016 (N = 5429)		
$lnMW_t$	-0.0856	-0.130	-0.005	-0.006
	(0.0687)	(0.0787)	(0.009)	(0.010)
		2010 to 2016 (N = 4205)		
$lnMW_t$	-0.1098	-0.171	-0.009	-0.011
	(0.0875)	(0.0966)	(0.011)	(0.012)
		2012 to 2016 (N = 2985)		
$lnMW_t$	-0.168*	-0.235**	-0.0131	-0.0147
	(0.0991)	(0.104)	(0.0121)	(0.0127)
State FE	Y	Y	Y	Y
Year Month FE	Y	Y	Y	Y

Notes. Table A.3 constructs job finding rate and separation rate as in Shimer (2012). The rows indicate the sample periods. I use state-clustered standard errors. *** means significant at 1% level, ** means significant at 5% level, * means significant at 10% level.

Table A.4: The Effect of Minimum Wages on Occupational Mobility

	(1) Age 16-30	(2) Age 30-45	(3) High School	(4) College	(5) 5 Lowest-Wage Occupations	(6) 5 Highest-Wage Occupations	(7) Age 16-30 $\times$ High School
					2004 to 2016 (N = 7956)		
$lnMW_t$	-0.009***	0.0003	-0.002	0.0004	-0.0030	-0.0007	-0.0070
	(0.0032)	(0.002)	(0.0025)	(0.0019)	(0.0021)	(0.0023)	(0.0046)
					2006 to 2016 (N = 6732)		
$lnMW_t$	-0.011***	-0.001	0.0047*	0.0009	-0.006**	0.0003	-0.011**
	(0.003)	(0.002)	(0.0025)	(0.0022)	(0.0023)	(0.0028)	(0.0046)
					2010 to 2016 (N = 4284)		
$lnMW_t$	-0.017***	-0.0028	-0.0096*	0.0006	-0.010***	-0.0007	-0.023**
	(0.005)	(0.0034)	(0.0048)	(0.0039)	(0.003)	(0.0044)	(0.0095)
					2012 to 2016 (N = 3060)		
$lnMW_t$	-0.017***	-0.001	-0.0060	0.0012	-0.0073*	0.0020	-0.020*
	(0.006)	(0.0044)	(0.0053)	(0.0040)	(0.0037)	(0.0053)	(0.0107)
State FE	Y	Y	Y	Y	Y	Y	Y
Year Month FE	Y	Y	Y	Y	Y	Y	Y

Notes. The columns present the results on the occupational mobility of the corresponding sub-groups. I construct occupational mobility using employer switching, i.e. an occupational code change counts as a switch only if there is an accompanying employer switching. I aggregate them at the state level and monthly frequency using final weight. Each row uses different sample periods. The controls are state-level monthly manufacturing and retail trade employment share. Table A.4 uses state-clustered standard errors. *** means significant at 1% level, ** means significant at 5% level, * means significant at 10% level.

A.3 Occupational Switching Via Unemployment

The occupational mobility construction in section 4 does not take into account occupational switch via unemployment. Workers can change occupations by going through unemployment. In this section, I measure occupational mobility by employment-unemployment-employment transitions and study its response to the minimum wage.

I merge three monthly files together. An occupational switch is identified if a worker 1. is employed in the first month, unemployed in the second month, and employment in the third month; 2. has different occupational codes in the first and third months. An occupational stayer is identified if a worker 1. is employed in the first month and the third month; 2. has the same occupational code in these two months.

The data allows for longer intervals for defining occupational mobility. For example, I could focus on the fourth and fifth month-in-sample and see if there is any change in the occupational code. However, the interval between these two periods is eight months, making it difficult to deduce whether an occupational code change across these two periods is truly an occupational switch. It is also difficult to interpret which kind of occupational mobility is affected because I do not observe what happens during the eight months. In section 4 I focus on the occupational mobility of employed workers while the measure described above focuses on the occupational mobility of workers who experience short-term unemployment spells.

I study the effect of the minimum wage on the occupational mobility of four subgroups: younger workers (16 to 30), older workers (30 to 45), younger and less-educated workers (younger workers with a high school degree or less), younger and low-wage workers (younger workers in 5 lowest wage occupations in table A.7).[32] I investigate the effect of the minimum wage in the first month denoted by $lnMW_t$ and the effect of three-month average minimum wages denoted by $\overline{lnMW}$. Table A.5 presents the results. Except for younger workers in low-wage occupations, the results are insignificant. This is likely because of poor data quality. For all sub-groups, over 50% of the observations are empty. The monthly average occupational mobility for younger workers is 0.7%, 0.3% for older workers, 0.9% for younger, less-educated workers, and 0.7% for younger workers in low-wage occupations.

The interpretation of the estimate for younger, less-educated workers is that a 10% first-month minimum wage increase decreases their occupational mobility by 10%. This points to evidence that the minimum wage dis-incentivizes occupational switch by short-term unemployment. I cannot deduce whether the decrease comes from reduced vacancy

[32] I do not consider older, more educated workers since only 10% of the observations are non-empty with the occupational mobility measure.

posting or wage compression.

Table A.5: The Effect of Minimum Wages on Occupational Mobility

	(1) Age 16-30	(2) Age 30-45	(3) Age 16-30 × High School	(4) Age 16-30 × 5 Lowest-Wage Occupations
$lnMW_t$	-0.0037	-0.0007	-0.0112	-0.017**
	(0.0046)	(0.0019)	(0.0076)	(0.0066)
$\overline{lnMW}$	-0.0035	-0.0012	-0.0112	-0.018***
	(0.0047)	(0.0019)	(0.0078)	(0.0069)
Observations	5508	5508	5508	5508
State FE	Y	Y	Y	Y
Year Month FE	Y	Y	Y	Y

Notes. Table A.5 identifies an occupational switch if and only if a worker is employed in the first period, unemployed in the second period, employed in the third period, and has different occupational codes in the two employment. An occupational stayer is a worker who has the same occupational code in the E-U-E transition. I aggregate the switchers and stayers at the state level and monthly frequency using final weight. $lnMW_t$ is the real minimum wage in the first month. $\overline{lnMW}$ is the three-month average minimum wage. Controls include state manufacturing and retail trade employment shares. Table A.5 uses state-clustered standard errors. *** means significant at 1% level, ** means significant at 5% level, * means significant at 10% level.

A.4 The Effect of the Minimum Wage on Detailed Occupational Transition Rates

Section 4 shows that the minimum wage decreases younger, less-educated workers' occupational mobility. In this subsection, I show that the minimum wage significantly reduces transitions from non-routine manual occupations (e.g. food preparation, building cleaning) to routine cognitive occupations (e.g. office and administrative support).

I first construct the detailed occupational transition rates. I restrict the sample to include only younger and less-educated workers. Occupational switching and occupational staying are measured the same as in section 4 using 4-digit level occupational codes. The transition matrix, however, uses four broad categories of occupations. In particular, I aggregate occupations into non-routine cognitive, non-routine manual, routine cognitive, routine manual occupations as in Autor and Dorn (2013). Doing so produces a 4-by-4 matrix for each state at the annual frequency. The sample period is 2004 to 2016. [33]

I merge the transition matrix with state-level minimum wages to examine the extent to which the minimum wage affects the transition rates. I use the two-way fixed effect model:

$$\text{Transition Rate}_{st} = \alpha + \beta lnMW_{st} + \delta_s + \lambda_t + \epsilon_{st} \tag{A.2}$$

The result in table A.6 shows that the minimum wage significantly decreases the transition from non-routine manual occupations to routine cognitive occupations. A 10% increase in the minimum wage decreases the transition rate from non-routine manual occupations to routine cognitive occupations by 0.04 percentage points or 4%. There is no significant effect on the other transition rates.

A.5 Low Wage Occupations Code

Table A.7: Low Wage Occupations 2010 SOC Code

Occupations	**2010 SOC Code**
Building and Grounds Cleaning	4200-4250
Personal Care and Service	4300-4650
Sales and Related	4700-4965
Office and Administrative Support	5000-5940
Transportation	9000-9420

A.6 The Minimum Wage and Mismatch

I show evidence that the minimum wage positively correlates with mismatch. The mismatch measure is from Guvenen et al. (2018) which I briefly introduce here. The NLSY79 records individuals' Armed Services Vocational Aptitude Battery (ASVAB) that provides detailed measures of occupation-relevant skills and abilities. It also has non-cognitive

[33] Using 2-digit level occupational codes or monthly frequency results in a large fraction of empty observations.

Table A.6: The Effect of Minimum Wages on Detailed Occupational Transition Rates

From \ To	Non-Routine Cognitive	Non-Routine Manual	Routine Cognitive	Routine Manual
Non-Routine Cognitive	-0.007	0.005	-0.001	0.009
	(0.006)	(0.008)	(0.012)	(0.007)
Non-Routine Manual	0.002	0.006	-0.011*	-0.003
	(0.003)	(0.006)	(0.006)	(0.005)
Routine Cognitive	-0.003	0.004	-0.004	-0.004
	(0.003)	(0.004)	(0.005)	(0.005)
Routine Manual	0.001	0.005	0.002	-0.011
	(0.003)	(0.003)	(0.005)	(0.009)
Observations	663	663	663	663
State FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y

Notes. Table A.6 shows the estimate results from equation (A.2). The numbers are coefficients of $lnMW_{st}$. For example, the number in the cell (Non-Routine Cognitive, Non-Routine Manual) is the coefficient of $lnMW_{st}$ in equation (A.2) using transition rate from non-routine cognitive occupations to non-routine manual occupations as the dependent variable. Equation (A.2) identifies an occupation switcher and a stayer the same as in section 4. Table A.6 uses state-clustered standard errors. *** means significant at 1% level, ** means significant at 5% level, * means significant at 10% level.

skills measures. The measures are collected at the beginning of the sample and hence reflect ex ante ability. Guvenen et al. (2018) aggregate the skills measures into three dimensions: math skills, verbal skills, and social skills. The skill composition data from O*NET specifies the intensity that each skill is applied in occupations. Guvenen et al. (2018) convert occupation skill composition into the three dimensions above. Mismatch is defined as the "distance" between ability and occupation skill composition.

The public version of the NLSY79 only has region information. Table A.8 shows the characterization of the regions and their average minimum wages. I aggregate the state-level minimum wages to region-level by unweighted averaging. I calculate real minimum wages using regional price index. I restrict the sample to include white and Hispanic workers between 16 to 30 with a high school degree as in section 4.

I use the following two-way fixed effect regression:

$$Mismatch_{irt} = \alpha + \beta lnMW_{rt} + X'_{irt}\gamma + \delta_t + \lambda_r + \epsilon_{irt} \tag{A.3}$$

Namely, I examine the correlation between individual-level mismatch and region-level real minimum wage changes. I include education, age, race, sex, and initial ability as controls. The estimate of the minimum wage is significant at the 10% level, as shown in the first column in table A.9. It suggests that a 10% increase in minimum wages is correlated with a 0.1 standard deviation increase in mismatch. When I restrict the sample to workers from 30 to 45, there is no significant correlation between the minimum wage and mismatch, as shown in the second column of table A.9.

I also consider estimating the effect differentially for males and females:

$$Mismatch_{irt} = \alpha + \beta_s lnMW_{rt} \times Sex + X'_{irt}\gamma + \delta_t + \lambda_r + \epsilon_{irt} \tag{A.4}$$

The results are in the third and fourth columns. The estimates of the minimum wage are similar for males and females. They are also consistent in magnitude with the results when I use equation (A.3).

Table A.8: States by Regions

Northeast (\$6.57)	North Central (\$6.33)	South (\$6.38)	West (\$6.54)
Connecticut	Illinois	Alabama	Alaska
Maine	Indiana	Arkansas	Arizona
Massachusetts	Iowa	Delaware	California
New Hampshire	Kansas	District of Columbia	Colorado
New Jersey	Michigan	Florida	Hawaii
New York	Minnesota	Georgia	Idaho
Pennsylvania	Missouri	Kentucky	Montana
Rhode Island	Nebraska	Louisiana	Nevada
Vermont	Ohio	Maryland	New Mexico
	South Dakota	Mississippi	Oregon
	North Dakota	North Carolina	Utah
	Wisconsin	Oklahoma	Washington
		South Carolina	Wyoming
		Tennessee	
		Texas	
		Virginia	
		West Virginia	

Notes. Table A.8 specifies the states that are in the four regions. The numbers in parenthesis are region-level average real minimum wage from 1979 to 2004.

Mismatch is also increasing in initial ability among both the young workers. The correlation between initial ability and mismatch is insignificant among older workers. This might be because of human capital accumulation with experience, making initial ability

Table A.9: Minimum Wage and Mismatch

	(1) Age 16-30	(2) Age 30-45	(3) Age 16-30	(4) Age 30-45
$lnMW_{rt}$	1.06*	0.09		
	(0.37)	(0.40)		
$lnMW_{rt} \times Male$			1.07*	0.10
			(0.38)	(0.39)
$lnMW_{rt} \times Female$			1.07*	0.08
			(0.40)	(0.40)
Age	0.02*	-0.01	0.02*	-0.01
	(0.01)	(0.01)	(0.01)	(0.01)
$White$	-0.03	-0.06	-0.03	-0.06
	(0.08)	(0.04)	(0.08)	(0.04)
$Education$	-0.04	-0.04*	-0.04	-0.04*
	(0.04)	(0.01)	(0.04)	(0.01)
$Female$	0.01	-0.02		
	(0.03)	(0.01)		
$Ability$	1.10***	-0.01	1.10***	-0.01
	(0.12)	(0.18)	(0.12)	(0.18)
Year Month FE	Y	Y	Y	Y
Region FE	Y	Y	Y	Y
N	13723	21356	13723	21356
R-squared	0.07	0.02	0.07	0.02

Notes. Table A.9 calculates real minimum wages using regional commodity price index from BLS. I use region-clustered standard errors. ** means significant at 5% level, * means significant at 10% level.

less correlated with mismatch.

A.7 Federal Minimum Wage States

Table A.10: Federal Minimum Wage States

Alabama	North Dakota
Georgia	Oklahoma
Idaho	South Carolina
Indiana	Tennessee
Kansas	Texas
Kentucky	Utah
Louisiana	Virginia
Mississippi	Wyoming

Notes. Table A.10 includes states that a have binding minimum wage equal to the federal minimum wage from 2008 to 2016.

B Proofs

Derivation of the value function.

Proof. The workers' problem is

$$V(x) = \mathbb{E}\left[\int_0^\tau e^{-rt} w(X_t) dt + e^{-r\tau} G(\tau)\right] \quad \text{subject to}$$
$$dX_t = \tilde{a} X_t dt + \sigma X_t dZ_t, \quad dX_{0-} = x$$

in which τ is a stopping time that corresponds to either switching occupations or unemployment and $G(\tau)$ is the outside option. To simplify notations, I denote $\left(\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j}) dH(\boldsymbol{j}) - \phi\right)$ by $\overline{V}$. Let τ_1 be the stopping time that the worker is separated exogenously. Let τ_2 be the stopping time that the worker receives outside offer. τ is then the minimum of τ_1 and τ_2. The worker's value function can be written further as

$$\begin{aligned} V(x) = \mathbb{E}[&\int_0^{h \wedge \tau_1 \wedge \tau_2} e^{-rt} w(X_t) dt + e^{-rh} V(X_h) \mathbb{I}_{\{h<\tau_1, h<\tau_2\}} \\ &+ e^{-r\tau_1} U \mathbb{I}_{\{\tau_1<h, \tau_1<\tau_2\}} + e^{-r\tau_2} \overline{V} \mathbb{I}_{\{\tau_2<h, \tau_2<\tau_1\}}] \end{aligned} \tag{B.1}$$

The symbol $\wedge$ means minimum of the two. Denote the generator of $\{X_t\}_{(t>0)}$ by $\mathcal{L}$, which is given by

$$\mathcal{L}V = \tilde{a} x V' + \frac{1}{2}\sigma^2 x^2 V''$$

Using the Ito's lemma on $V(X_h)$, we have

$$V(X_h) = v(x) + \int_0^h (\mathcal{L}V)(X_t) dt + \text{local martingale}$$

Plug this back into equation (B.1), we have

$$\begin{aligned} \left(1 - e^{-rh}\right) V(x) = \mathbb{E}[&\int_0^{h \wedge \tau_1 \wedge \tau_2} e^{-rt} (w + \mathcal{L}V)(X_t) dt \\ &+ e^{-r\tau_1} (U - V(x)) \mathbb{I}_{\{\tau_1<h, \tau_1<\tau_2\}} + e^{-r\tau_2} (\overline{V} - V) \mathbb{I}_{\{\tau_2<h, \tau_2<\tau_1\}}] \end{aligned} \tag{B.2}$$

Note that because τ_1 and τ_2 are independent,[34] we have

$$\mathbb{I}_{\{\tau_1<h, \tau_1<\tau_2\}} = (1 - e^{\delta h}) e^{\alpha\lambda h}$$
$$\mathbb{I}_{\{\tau_2<h, \tau_2<\tau_1\}} = (1 - e^{\alpha\lambda h}) e^{\delta h}$$

[34] To construct such independent stochastic processes, see e.g. Rogers and Williams (2000).

Divide equation (B.2) by $\frac{1}{h}$ and let $h \to 0$, I arrive at equation (2). □

Lemma B.1. *The wage function has the form*

$$w(x) = \beta x + (1-\beta)b + \lambda(1-\beta)(1-\alpha\mathbb{I}_{sw})\left[\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j})dH(\boldsymbol{j}) - \phi - U\right] \tag{B.3}$$

Proof. I seek the classic solution of equation (2). By the generalized Nash bargaining, we have

$$\begin{aligned} \beta J(x) &= (1-\beta)(V(x) - U) \\ \beta J'(x) &= (1-\beta)V'(x) \\ \beta J''(x) &= (1-\beta)V''(x) \end{aligned} \tag{B.4}$$

Now let us look at the value functions $V(x)$ and $J(x)$. For simplicity, I suppress the dependence on x for functions $V(x)$ and $J(x)$ and write V and J. Using notation from above, I write the integral $\int_{\mathbb{T}^n} V(x_p, \boldsymbol{j})dH(\boldsymbol{j}) - \phi$ as $\overline{V}$. Then the value functions are

$$\begin{aligned} (1-\beta)r(V-U) =& (1-\beta)w(x) + (1-\beta)V'\tilde{a}x + \frac{1}{2}(1-\beta)\sigma^2x^2V'' \\ & + (1-\beta)\alpha\lambda\mathbb{I}_{sw}\left(\overline{V} - V\right) - \delta(1-\beta)(V-U) - r(1-\beta)U \\ \beta r J =& \beta x - \beta w(x) + \beta J'\tilde{a}x + \frac{1}{2}\beta\sigma^2x^2J'' - \beta\alpha\lambda\mathbb{I}_{sw}J - \delta\beta J \end{aligned}$$

Using the relations in equation (B.4), we can subtract the first equation by the second one and solve for $w(x)$ to get

$$w(x) = \beta x + (1-\beta)b + (1-\beta)\lambda(1-\alpha\mathbb{I}_{sw})\left(\overline{V} - U\right)$$

□

Proof of proposition 1.

Proof. I solve the differential equation (4) in the interval $(\underline{x}, x_s)$. In this interval, the differential equation can be written as

$$(r+\delta)J - \tilde{a}xJ' - \frac{1}{2}\sigma^2x^2J'' - (1-\beta)x + (1-\beta)(b + \lambda A(m)) = 0 \tag{B.5}$$

where $A(m)$ is some constant depending on the minimum wage. Define $f(x,m) \equiv (1-\beta)x - (1-\beta)(b+\lambda A(m))$. The general solution to equation equation (B.5) is

$$J(x) = C_0^0 x^{\gamma_0^0} + C_1^0 x^{\gamma_1^0} \tag{B.6}$$

The parameters γ_0^0 and γ_1^0 can be calculated directly via

$$\begin{aligned} \gamma_0^0 &= -\frac{\tilde{a}}{\sigma^2} + \frac{1}{2} - \sqrt{(\frac{1}{2} - \frac{\tilde{a}}{\sigma^2})^2 + \frac{2(\delta+r)}{\sigma^2}} < 0 \\ \gamma_1^0 &= -\frac{\tilde{a}}{\sigma^2} + \frac{1}{2} + \sqrt{(\frac{1}{2} - \frac{\tilde{a}}{\sigma^2})^2 + \frac{2(\delta+r)}{\sigma^2}} > 0 \end{aligned} \tag{B.7}$$

Equation (B.5) also admits a special solution

$$A(m,x) = \frac{2}{\sigma^2(\gamma_1^0 - \gamma_0^0)}\left[x^{\gamma_1^0}\int_x^{+\infty} s^{-\gamma_1^0 - 1} f(s,m)ds + x^{\gamma_0^0}\int_0^x s^{-\gamma_0^0-1} f(s,m)ds\right] \tag{B.8}$$

where $f(x,m)$ is a function depending on the minimum wage. Specifically, when the wage function is smaller than the minimum wage, $f(x,m) = m$. The solution to equation (B.5) hence has the form

$$J(x) = C_0^0 x^{\gamma_0^0} + C_1^0 x^{\gamma_1^0} - A(m,x) \tag{B.9}$$

Solving for $A(m,x)$, I arrive at equation (8) for $J(x)$ in $(\underline{x}, x_s)$. Similarly, I can solve for the value function in $(x_s, +\infty)$. The power coefficients of solution γ_0^1 and γ_1^1 are given by

$$\begin{aligned} \gamma_0^1 &= -\frac{\tilde{a}}{\sigma^2} + \frac{1}{2} - \sqrt{(\frac{1}{2} - \frac{\tilde{a}}{\sigma^2})^2 + \frac{2(\alpha\lambda+\delta+r)}{\sigma^2}} < 0 \\ \gamma_1^1 &= -\frac{\tilde{a}}{\sigma^2} + \frac{1}{2} + \sqrt{(\frac{1}{2} - \frac{\tilde{a}}{\sigma^2})^2 + \frac{2(\alpha\lambda+\delta+r)}{\sigma^2}} > 0 \end{aligned} \tag{B.10}$$

□

Lemma B.2. *There are boundary conditions such that the value function equation (8) is strictly increasing.*

Proof. To see this, I abstract from the other boundary conditions and consider only $J(\underline{x}) = 0$ and $J(x_s) = B$. This define a mapping from $(\underline{x}, B)$ to (C_0^0, C_1^0). Fix $x_s > 1$ and let $B \longrightarrow +\infty$, it must be the case that $C_1^0 \longrightarrow +\infty$, otherwise the function remains bounded in the interval $(\underline{x}, x_s)$. C_1^0 can be made so large that $J(x)$ is increasing on $(\underline{x}, x_s)$ because we can choose x_s so that the first term remains bounded. This completes the proof. □

Lemma B.3. *The threshold x_s characterizes the workers' on-the-job search decision.*

Proof. If x_s is unique then this is clearly the case. By lemma B.2, the value function can be chosen to be monotonic. Under such a condition, x_s is unique and the workers search on the job if their output is between $\underline{x}$ and x_s. $\square$

Proof of proposition 2.

Proof. I utilize tools from functional analysis to prove the proposition. I make the ability and the occupation distribution to be more general so that it is given by a joint CDF $N(\boldsymbol{a}, \boldsymbol{j})$ instead of independently by $G(\boldsymbol{a})$ and $H(\boldsymbol{j})$. To prove proposition 2, I need one more assumption:

Assumption: The joint CDF $N(\boldsymbol{a}, \boldsymbol{j})$ has a continuous pdf $n(\boldsymbol{a}, \boldsymbol{j})$.

The problem at hand can be restated in the following abstract form:

1. Denote the constant in the ODE equation (2) by $f(\boldsymbol{a})$. That is, I define $f(\boldsymbol{a}) \equiv \int_{\mathbb{T}^n} V(x_p, \boldsymbol{a}, \boldsymbol{j}) n(\boldsymbol{a}, \boldsymbol{j}) d\boldsymbol{j}$. I let the function to be in the Banach space $L^1(\mathbb{T}^n)$. That is, the function is integrable on $\mathbb{T}^n$. Given a value $f(\boldsymbol{a})$, there is a corresponding solution of the value function $V(x, \boldsymbol{a}, \boldsymbol{j})$ that is twice continuously differentiable. I denote $\mathbb{V}(\boldsymbol{a})$ to be the $L^1([0, \overline{x}] \times \mathbb{T}^n)$ valued function $V(\cdot, \boldsymbol{a}, \cdot)$. By the uniqueness of the solution, I can define a mapping T that maps from $L^1(\mathbb{T}^n)$ to $L^1([0, \overline{x}] \times \mathbb{T}^n)$:

$$Tf = \mathbb{V} \tag{B.11}$$

   This mapping is bounded by standard result in the ODE literature. By the uniqueness of solution and linearity of the differential operator, it is also linear.

2. Having obtained a family of solutions $\{V\}_{(\boldsymbol{a}, \boldsymbol{j})}$, I fix a point x so that the family of solutions is mapped into a family of functions on $\mathbb{T}^n \times \mathbb{T}^n$. I denote this mapping by P:

$$PV = V(x, \cdot, \cdot) \tag{B.12}$$

   This mapping is clearly linear and bounded:

$$P(V_1 + V_2) = V_1(x, \cdot, \cdot) + V_2(x, \cdot, \cdot) \tag{B.13}$$

3. I integrate the function obtained in step 2 by the CDF $N(\boldsymbol{a}, \boldsymbol{j})$ with respect to $\boldsymbol{j}$.

Denote this mapping Q:

$$QV(x, \boldsymbol{a}, \boldsymbol{j}) = \int_{\mathbb{T}^n} V(x, \boldsymbol{a}, \boldsymbol{j}) n(\boldsymbol{a}, \boldsymbol{j}) d\boldsymbol{j} \equiv g(\boldsymbol{a}) \tag{B.14}$$

The end result is that I map the function $f(\boldsymbol{a})$ into the function $g(\boldsymbol{a})$ by the operators T, P, Q. That is,

$$QPT \circ f = g \tag{B.15}$$

Note that all three operators Q, P, T are linear and bounded. Let us denote the composite mapping by R. The problem reduces to finding a fixed point of the mapping R so that

$$Rf = f \tag{B.16}$$

This can be done by observing that the operator Q is a compact operator. For a proof, see Lax (2002). Since P and T are also linear bounded operators, the composite map R is compact. This mapping is non-trivial, so there is at least one non-zero eigenvalue ξ with eigen-function f such that $Rf = \xi f$. I could now choose the job finding rate to be ξ^{-1} so that $R'f = f$.

□

Proof of proposition 3.

Proof. I solve the equation equation (9) in the interval $(\underline{x}, x_s)$. The corresponding ODE is

$$\frac{\sigma^2}{2} x^2 f''(x) + (2\sigma^2 - \tilde{a}^2) x f'(x) + (\sigma^2 - \tilde{a} - \delta) f(x) = 0 \tag{B.17}$$

Similar to the proof of proposition 1, the general solution is

$$f(x) = B_0^0 x^{\eta_0^0} + B_1^0 x^{\eta_1^0} \tag{B.18}$$

in which the parameters η_0^0 and η_1^0 are given by

$$\begin{aligned} \eta_0^0 &= -\frac{2\sigma^2 - \tilde{a}}{2} + \frac{1}{2} - \sqrt{(\frac{1}{2} - \frac{2\sigma^2 - \tilde{a}}{2})^2 + \frac{2(\tilde{a} + \delta - \sigma^2)}{\sigma^2}} < 0 \\ \eta_1^0 &= -\frac{2\sigma^2 - \tilde{a}}{2} + \frac{1}{2} + \sqrt{(\frac{1}{2} - \frac{2\sigma^2 - \tilde{a}}{2})^2 + \frac{2(\tilde{a} + \delta - \sigma^2)}{\sigma^2}} > 0 \end{aligned} \tag{B.19}$$

Similarly I can solve for $f(x)$ in the interval $(x_s, \overline{x})$ and calculate that

$$\begin{aligned}\eta_0^1 &= -\frac{2\sigma^2 - \tilde{a}}{2} + \frac{1}{2} - \sqrt{(\frac{1}{2} - \frac{2\sigma^2 - \tilde{a}}{2})^2 + \frac{2(\tilde{a} + \delta + \alpha\lambda - \sigma^2)}{\sigma^2}} < 0 \\ \eta_1^1 &= -\frac{2\sigma^2 - \tilde{a}}{2} + \frac{1}{2} + \sqrt{(\frac{1}{2} - \frac{2\sigma^2 - \tilde{a}}{2})^2 + \frac{2(\tilde{a} + \delta + \alpha\lambda - \sigma^2)}{\sigma^2}} > 0\end{aligned} \tag{B.20}$$

This completes the proof. □

Proof of proposition 5.

Proof. Let us first formulate the worker's problem in a different way. The worker chooses an increasing sequence of stopping times, τ_n, $\tau_n \to +\infty$, representing the decision of when to search on the job and when to quit to unemployment, and $\boldsymbol{j}_n$ is the occupational the worker wants to switch to or the state of unemployment. Let $p = \{\tau_n, \boldsymbol{j}_n\}_{n \in \mathbb{N}}$, the worker's problem can be written as:

$$V(x) = \sup_p \mathbb{E}\left[\int_0^{+\infty} e^{-rt} w(X_t^{x_s, \boldsymbol{j}}, \boldsymbol{j}) dt - \sum_{n=1}^{\infty} e^{-r\tau_n} \phi\right] \tag{B.21}$$

where $w(x, \boldsymbol{j})$ is the payoff function in occupation $\boldsymbol{j}$, in this case given by equation (6). $X_t^{x_s, \boldsymbol{j}}$ is the output process when the worker is in occupation $\boldsymbol{j}$ with the initial output x_s. $\sum_{n=1}^{\infty} e^{r\tau_n} \phi$ is the sum of the discounted switching cost.

It can be shown that the solution is given by the variational inequality

$$\min\left[rV_{\boldsymbol{i}} - \tilde{a}xV_{\boldsymbol{i}}' - \frac{1}{2}\sigma^2 x^2 V_{\boldsymbol{i}}'' - w(x), V_{\boldsymbol{i}} - \int_{\mathbb{T}^n} V_{\boldsymbol{j}} dH(\boldsymbol{j}) + \phi\right] = 0 \tag{B.22}$$

$V_{\boldsymbol{i}}$ is the value function on occupation indexed $\boldsymbol{i}$, similarly for $V_{\boldsymbol{j}}$. There is an output level x_m such that if $x < x_m$, $w(x)$ is equal to the minimum wage m. Clearly, x_m is increasing m. It is also straight forward to see that when $x < x_m$, $\partial V_{\boldsymbol{i}}/\partial m > 0$: solving the ODE when $x < x_m$ we get that the minimum wage enters the value function linearly. By the smooth pasting condition and the uniqueness of the solution, this implies that $\partial V_{\boldsymbol{i}}/\partial m > 0$ everywhere. The interpretation of the result is that conditional on employment, workers' value function is increasing in the minimum wage.

The increase in the value function because of the minimum wage is decreasing in the drift $\tilde{a}$ because $\partial^2 V_{\boldsymbol{i}}/\partial m \partial \tilde{a} < 0$. The result comes from solving for the value function when $w(x) = m$, in which the solution is linear in m with multiply coefficient equal to

$2/\sigma^2(\gamma_1^0 - \gamma_0^0)$. Taking the derivative of this multiply coefficient with respect to $\tilde{a}$ shows that it is decreasing in $\tilde{a}$, or the difference $(\gamma_1^0 - \gamma_0^0)$ is increasing in $\tilde{a}$. Mismatch decreases $\tilde{a}$, leading to a larger positive effect of the minimum wage on workers' value function. The rest of the proof is the same as the one shown in section 3. $\square$

C Numerical Details

I present the details of the model estimation in this section. I discretize the worker ability distribution so that there are ten levels of ability: $(0.1, 0.2, ..., 1)$. I do the same for the occupation distribution. I pool 2008 to 2017 CPS Merged Outgoing Rotation Groups (MORG) data to be the sample. In this sample, 29.1% of the workers complete college degree and above, 28.6% of the workers have associate degree or vocational training, and 42.3% of the workers have high school degree or less. I set grid 1 and 4 to be the low ability workers, 5 to 7 to be the medium ability workers, and 8 to 10 to be the high ability workers. I calibrate the parameters of the Beta distribution to match the empirical composition of the workers by education exactly. The resulting parameterization is $Beta(0.8879, 0.9414)$.

As mentioned in section 5, I introduce the search accuracy parameter ρ which determines the probability that a worker is sorted into her optimal occupation. The joint distribution of ability and occupation is hence implied by the distribution of workers $Beta(0.8879, 0.9414)$ and ρ. ρ impacts the occupational mobility rate, the effect of minimum wages on occupational mobility, and wage gain from switching occupations. The main target for ρ is the 1% wage gain from switching calculated by Guvenen et al. (2018).

An important parameter in the model is the initial output x_p. I set it as a function of ability a:

$$x_p(a) = c_0 + c_1 a \tag{C.1}$$

I set c_0 to be 5.35 and c_1 to be 15 to match the empirical wage distribution.[35] Both the on-the-job-search cutoff equation (17) and the endogenous separation cutoff equation (18) are fractions of the initial output.

I follow Lise and Robin (2017) and use GMM to estimate the parameters. In constructing the variance-covariance matrix of the vector of moments, the moments consist of only means. I use the Newey-West estimator to estimate the variance-covariance matrix as in Lise and Robin (2017) and choose the lag order to be 6.

The moment targets and the corresponding model estimates are listed in table C.1. The model matches the moment targets quite well given the heterogeneity. Specifically, the employment elasticity and the occupational mobility elasticity are matched quite well which are the main focus of the paper.

In the GMM estimation, I set weights to be 100 for moments 5 to 13. The weights for the other moments are set to be 1.

[35] I need to modify the proof of proposition 2 to adapt for the case in which initial output is an affine function of ability rather than fixed. When I have discrete distribution of abilities and occupations this can be done easily.

Table C.1: Moment Targets and Model Estimates

Targets	Data	Model Estimates
Wage gain	1%	1.6%
Separation rate, low ability workers	7.5%	5.3%
Separation rate, mid ability workers	6.2%	5%
Separation rate, high ability workers	3.6%	3.6%
Fraction of workers earning less than $7.25	5%	4.6%
Fraction of workers earning less than $15	40%	41%
Occupational mobility, low ability workers	2.6%	3.8%
Occupational mobility, mid ability workers	1.5%	1.9%
Occupational mobility, high ability workers	1.1%	1%
Elasticity of occupational mobility, low ability workers	-0.3	-0.3
Elasticity of occupational mobility, mid ability workers	0	-0.1
Elasticity of occupational mobility, high ability workers	0	0
Elasticity of employment, low ability workers	-0.1	-0.1
Elasticity of employment, mid ability workers	0	0
Elasticity of employment, high ability workers	0	0
P20/P10	1.21	1.24
P30/P10	1.46	1.47
P40/P10	1.75	1.73
P50/P10	2.06	2.02
Variance to mean ratio	13	13

In section 5 I decompose the effect of minimum wages on occupational mobility and aggregate output by looking at the wage compression channel alone and by looking at the overall effect. I obtain the results without employment effect by setting p_3 to be 0 and λ' to be 0.36. p_3 governs the displacement effect of the minimum wage while λ' governs the effect on vacancy posting.

I present here the results with *only employment effect*. To do this, I set the parameter s_3 to be 0, which determines the response of occupational mobility to the minimum wage. The occupational mobility for low ability workers decreases by 3%, and overall occupational mobility decreases by 2%. The effects are almost additive: when I restrict the model to have no employment effect, the corresponding estimates are 42% and 28% while the results with both channels are 44% and 30% respectively.

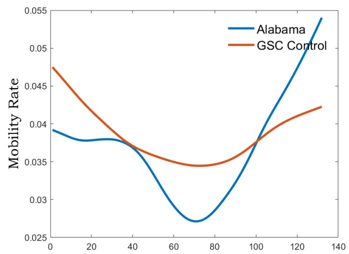
Turning to the effect on aggregate output, when I restrict the model to have only employment effect, the estimate shows that aggregate output decreases by 0.15% when the minimum wage increases to $15. This estimate is insignificant at 5% level.

D Extra Figures

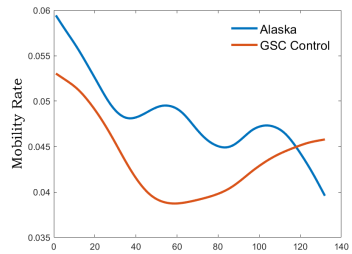
D.1 States and Their GSC Controls

Figure D.1: GSC Fit

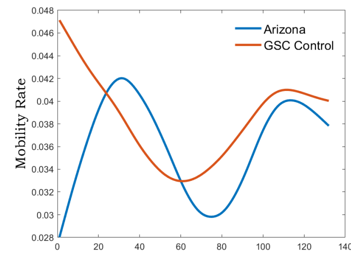
(a) Alabama



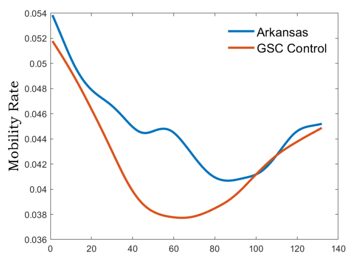
(b) Alaska



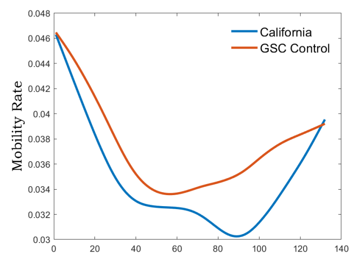
(c) Arizona



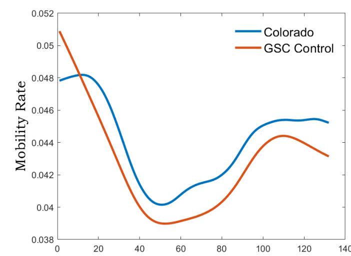
(d) Arkansas



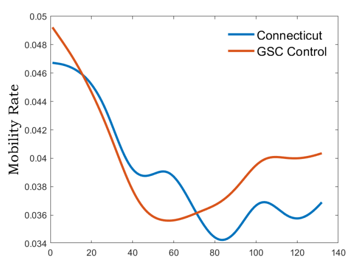
(e) California



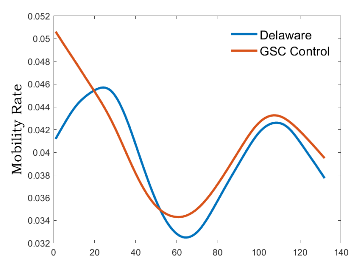
(f) Colorado



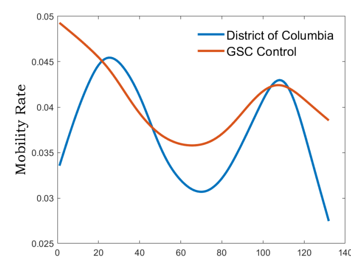
(g) Connecticut



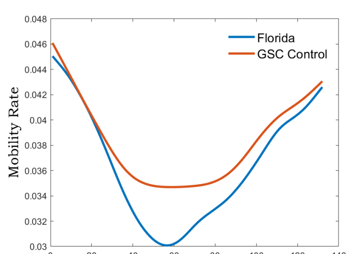
(h) Delaware



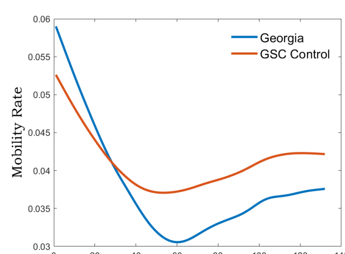
(i) District of Columbia



(j) Florida



(k) Georgia



(l) Hawaii

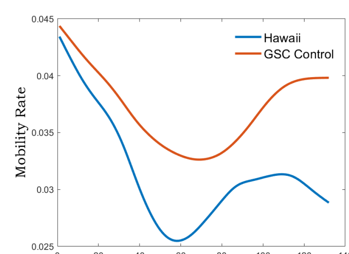
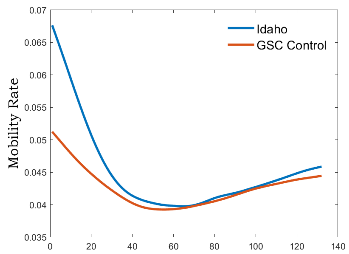
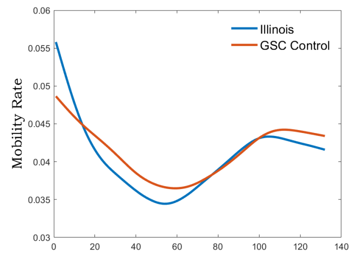


Figure D.2: GSC Fit

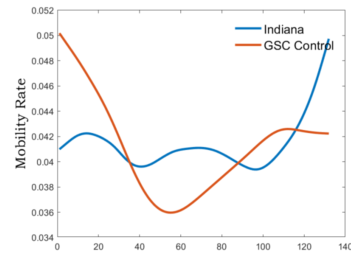
(a) Idaho



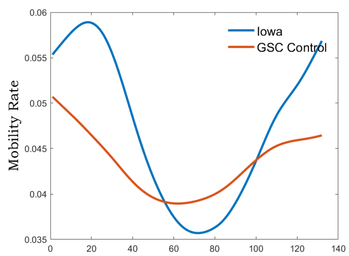
(b) Illinois



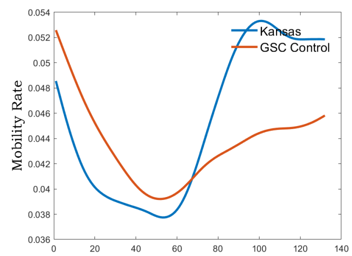
(c) Indiana



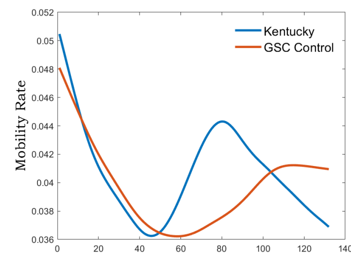
(d) Iowa



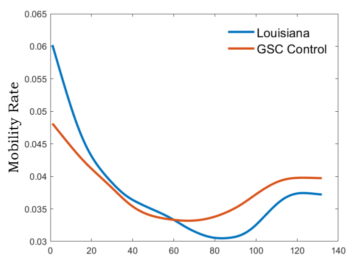
(e) Kansas



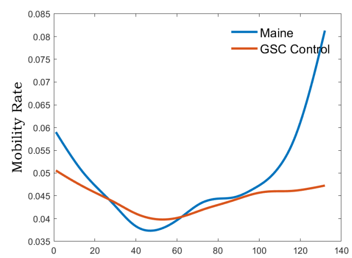
(f) Kentucky



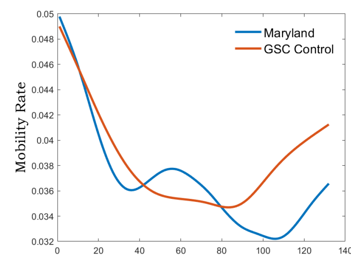
(g) Louisiana



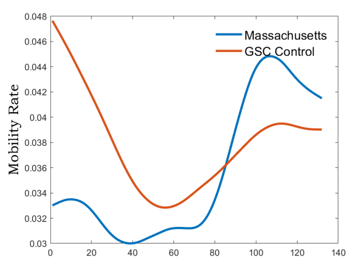
(h) Maine



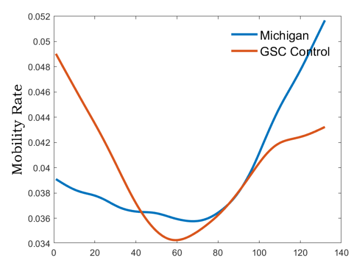
(i) Maryland



(j) Massachusetts



(k) Michigan



(l) Minnesota

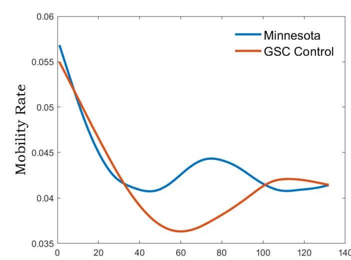
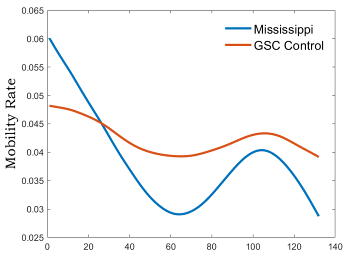
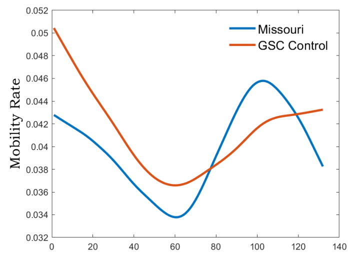


Figure D.3: GSC Fit

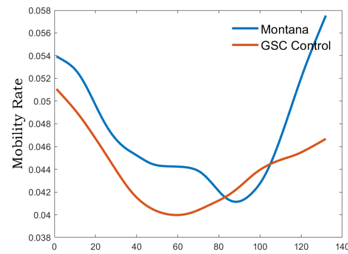
(a) Mississippi



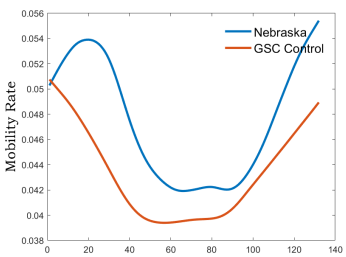
(b) Missouri



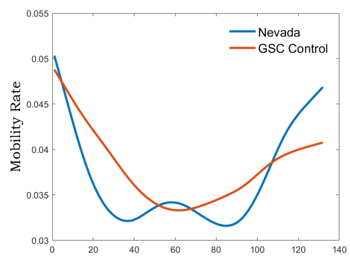
(c) Montana



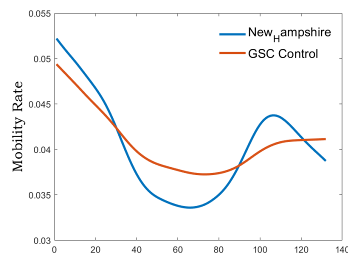
(d) Nebraska



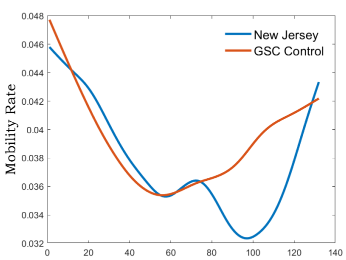
(e) Nevada



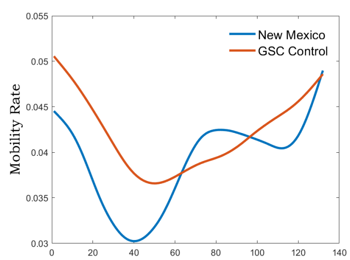
(f) New Hampshire



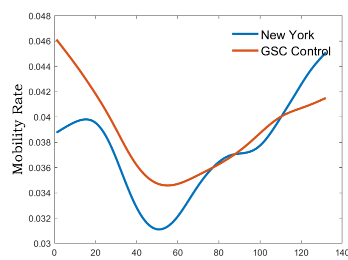
(g) New Jersey



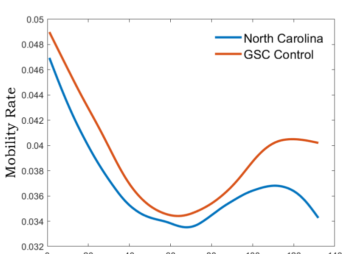
(h) New Mexico



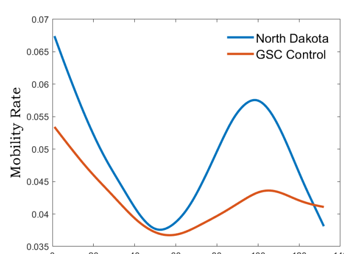
(i) New York



(j) North Carolina



(k) North Dakota



(l) Ohio

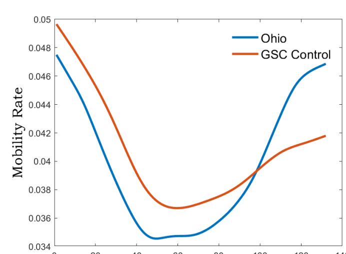
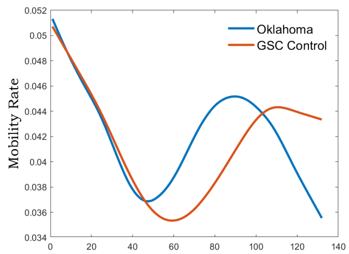
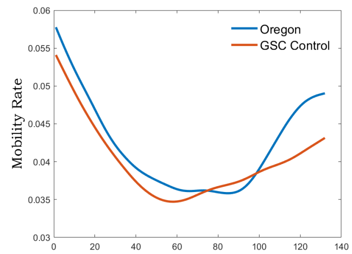


Figure D.4: GSC Fit

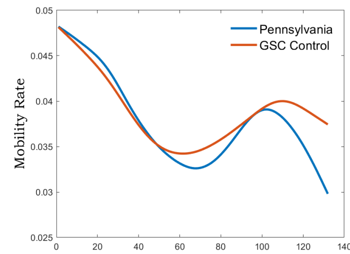
(a) Oklahoma



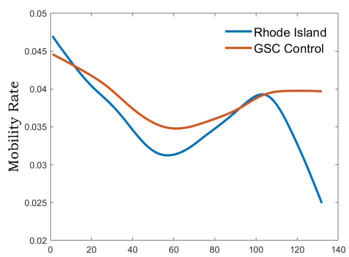
(b) Oregon



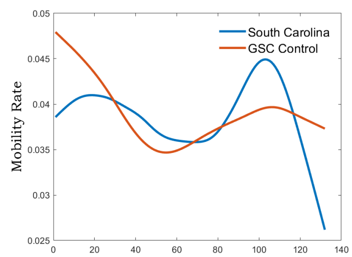
(c) Pennsylvania



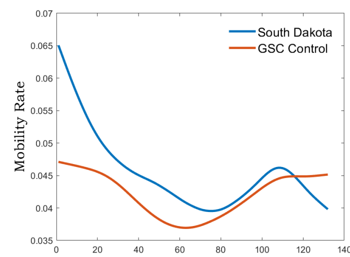
(d) Rhode Island



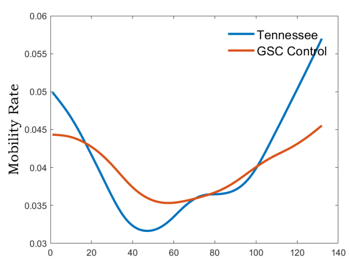
(e) South Carolina



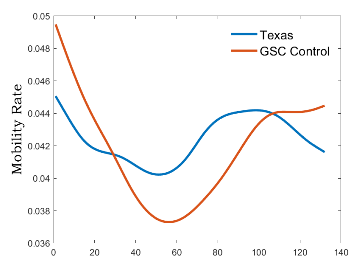
(f) South Dakota



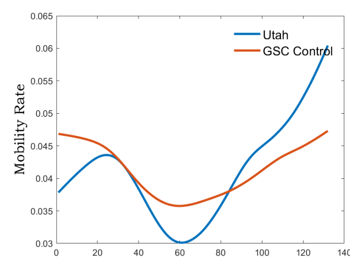
(g) Tennessee



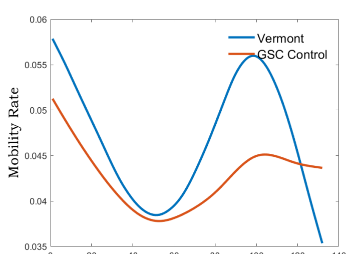
(h) Texas



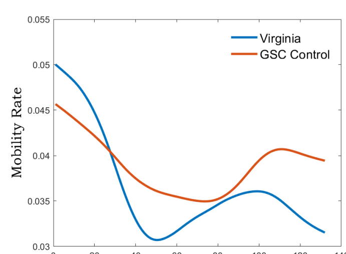
(i) Utah



(j) Vermont



(k) Virginia



(l) Washington

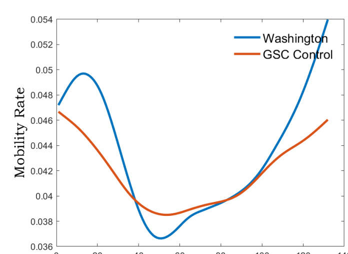
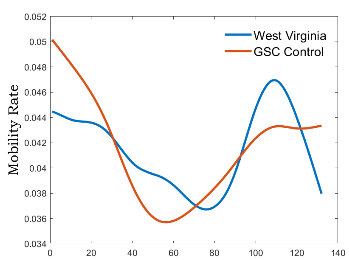
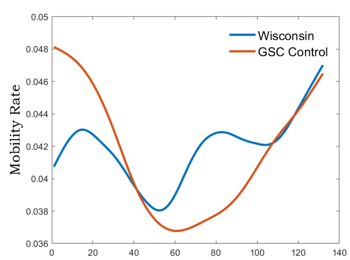


Figure D.5: GSC Fit

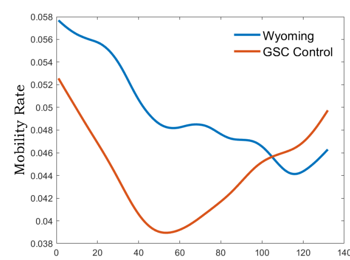
(a) West Virginia



(b) Wisconsin



(c) Wyoming



E Micro-Found the Productivity Process

I micro-found the productivity process in section 2. Consider a Ben-Porath economy with human capital accumulation and labor supply decisions. Occupation-specific human capital determines productivity. To simplify, I assume that the match lasts for T periods and I normalized the outside option to be 0. One can think of T as a stopping time which does not affect the result in this section. The worker's objective is to minimize the disutility of labor supply and human capital accumulation, with initial human capital equal to h_1.

$$\min_{\{h_{t+1}\}} \sum_{t=1}^{T} \beta^t D(l_t, s_t) \tag{E.1}$$

The effective labor supply, l_t is decreasing in human capital h_t. A worker can accumulate human capital by exerting effort s_t. I assume that the effective labor supply is given by

$$l_t = g(h_t) \tag{E.2}$$

The function $g(h_t)$ is continuously differentiable and $g' < 0$. The interpretation is that human capital reduces labor disutility. Workers' effort leads to human capital growth subject to some shock ϵ_t.

$$h_{t+1} = h_t + f(\tilde{a}, \epsilon_t, s_t) \tag{E.3}$$

Equation (E.3) suggests that human capital accumulation depends on match-specific factor $\tilde{a}$, effort s_t, and some shock. I assume $\partial^2 f/\partial\epsilon_t\partial s_t > 0$, so that with good shocks, the same level of effort raises human capital by more. This is in line with the literature on the persistent effect of labor market entry shocks on wages, in which one explanation is that workers accumulate human capital faster with good entry shocks (see e.g. Oreopoulos et al. (2012)). In addition, I assume $\partial^2 f/\partial\tilde{a}\partial s_t > 0$, so the marginal gain in human capital by effort is increasing in match quality $\tilde{a}$. The shock ϵ has pdf $\phi(\epsilon)$.

The disutility function is separable in effective labor supply and effort.

$$D(l_t, s_t) = d_1(l_t) + d_2(s_t) \tag{E.4}$$

The recursive formulation of the problem is hence

$$V(h_t, \epsilon_t) = \min_{s_t} d_1(g(h_t)) + d_2(s_t) + \beta \int V(h_t + f(\tilde{a}, \epsilon_t, s_t), \epsilon_{t+1})\phi(\epsilon_{t+1})d\epsilon_{t+1} \tag{E.5}$$

The worker chooses the human capital accumulation effort s_t to trade off current disutility

$d_2(s_t)$ with future labor disutility reduction. The first order condition reflects this trade-off:

$$d_2'(s_t) + \beta \int \frac{\partial V}{\partial h}(h_t + f(\tilde{a}, \epsilon_t, s_t), \epsilon_{t+1})\phi(\epsilon_{t+1})d\epsilon_{t+1} = 0 \tag{E.6}$$

This is an implicit function of s_t, which implies that the human capital increment is determined by $f(\tilde{a}, \epsilon_t, s_t(h_t, \epsilon_t))$. Note that the envelop condition gives

$$\frac{\partial V}{\partial h}(h_t, \epsilon_t) = d_1'(g(h_t))g'(h_t) + \beta \int \frac{\partial V}{\partial h}(h_{t+1}, \epsilon_{t+1})\phi(\epsilon_{t+1})d\epsilon_{t+1} \tag{E.7}$$

With equation (E.7), it is easy to verify that $\partial^2 f/\partial h_t \partial \epsilon_t > 0$ and $\partial^2 f/\partial h_t \partial \tilde{a} > 0$. Using Taylor expansion, I have

$$f(\tilde{a}, \epsilon_t, h_t) \approx \beta_0 + \beta_1 \tilde{a} + \beta_2 \epsilon_t + \beta_3 h_t + \beta_4 \tilde{a} h_t + \frac{\beta_5}{\sigma}\sigma \epsilon_t h_t \tag{E.8}$$

in which $\beta_4, \beta_5 > 0$. I also assume that shock per se does not affect human capital accumulation so that $\beta_2 = 0$.

Plug equation (E.8) into equation (E.3). With some normalization, I arrived at

$$h_{t+1} = h_t + (\tilde{a} + \sigma \epsilon_t)h_t \tag{E.9}$$

This is equivalent to

$$\frac{h_{t+1} - h_t}{h_t} = \tilde{a} + \sigma \epsilon_t \tag{E.10}$$

Equation (E.10) is the same as equation (1). The result suggests that in a Ben-Porath economy with endogenous human capital accumulation, the human capital process can evolve stochastically, depending on match-specific component $\tilde{a}$ and idiosyncratic shock ϵ.